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IS THE UNIVERSE PLANNED? *

BY

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The address which I am about to deliver was prompted by our esteemed Chairman of the Bartol Research Foundation Committee, and it is with gratitude for his friendship, for our long association, and for his continued advice and support that I now give the address in his honor.

Dr. Clamer had just read a book by Roy Chapman Andrews entitled *Nature's Ways*, a book pointing out how Nature has, in a sense, anticipated many of the subsequent inventions of man, not, of course, in exact form as regards mechanism, but in close analogy as regards principle. On reading this book, Dr. Clamer felt strongly the urge to inquire as to the meaning of it all, what inferences are to be drawn from the picture as regards the concept of a universe founded upon intelligent design and guidance, and what evidence there is as to the nature of man's place in it. Dr. Clamer expressed a desire that in my address this year I discuss these things and bring to bear upon the story such evidence as science has provided.

Now in developing the consequences of this matter, I must admit no barriers in the paths of thought. If, in what I have to say, I offend the convictions of some by minimizing the element of personality inherent in the fundamental doctrine of Divine guidance, I apologize. If I offend others by implying a suspicion of too strong a sensitivity to an admission of such guidance, so that I appear no longer as a man of science, but as an ecclesiastical philosopher—which picture of me will,

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I am sure, cause many of my friends to smile—again I apologize. And so I apologize to both extremes for the hurt which my words may do to their feelings; but, for daring to think of these things, I owe no man an apology, for the brain of man is the only apparatus he has for the appraisal of that which is around him; and, even should he come to accept certain beliefs on the basis of blind faith, it is his brain which must have been called upon to make the decision which constitutes, as among different kinds of faith, the choice of that one to which he will adhere.

On the screen is a picture[2] of what appears to be a mass of rubble. It has no beauty. It seems to have no design, and appears but as the waste product of something which once had greater significance.

Suppose that as you wandered in the wilderness, you came across such a mass of rubble. I believe that you would not be intrigued by contemplating it. Suppose, however, that on separating its parts and magnifying them so that you can see their structure, you find that the rubble is composed of that which I now have upon the screen. I think you will begin to be intrigued by certain regularities of form which are now evident, and you will wonder whether the rubble is as formless and purposeless as it seemed to be.

Suppose that as you continue your journey you come upon this structure.[3] And suppose that as you examine the cathedral you find that, in the last analysis, it has been built out of just such fundamental forms as you had found when you examined the contents of the mass of rubble. I think that now you will begin to feel really intrigued about the matter and will be unable to refrain from asking whether the pieces of that apparently formless mass were not intended, somehow or other, and in due time, to be fitted together so as to build a Notre Dame. Yet, as you view this rubble, and search further in the territory where it lies, you find no architect to whom you can attribute the design of these fundamental building blocks, and no builder who seemed to have been responsible for the cathedral. Perhaps the architect and his builders have died after the original plan had been brought to fruition. This might seem the simplest solution for the moment. Suppose, however, that on returning to this territory at a later date you find new cathedrals and other structures all welding themselves together into a richer community and all built from the same kind of fundamental shapes. I think you will be tempted to inquire how it all came about.

There will be those who tell you that they have come to believe in the existence of invisible architects and builders who, silently, invisibly, and by means unknown to man have gradually developed what you see around you and who indeed continue to develop it to groups of struc-

[2] The several pictures mentioned in the lecture have not been reproduced in the JOURNAL, since the text is fully explanatory.

[3] Here was shown a picture of Notre Dame in Paris.

tures of greater and greater magnificence. These people will hold the invisible builders and architects in awe, and in greatest awe will they hold the master architect who directs the activities of all of them.

And then there will be another group of people who disclaim all belief in these invisible beings and who will tell you that once there was a great storm which blew the masses of rubble about, so that, when they settled, they settled by accident in such a way as to form the buildings which you see around you. However, I think you will feel a little dissatisfied by the explanation which has been given you, and even if you are willing to accept it, you will be inclined to ask how and why the fundamental shapes originally in the rubble had acquired such forms as to make it possible to build these structures out of them, regardless of whether the structures were built by invisible builders, as maintained by some, or as an accidental result of the winds, as maintained by others.

If you talk with the first group, you will find that, prompted by the veneration in which they hold the invisible architects and builders, they have created for them all sorts of awe-inspiring characteristics, apart from those involved in the operation of the building. They will have created elaborate rituals in which these beings function and, for that master builder whom they call God, they will have imagined all sorts of characteristics, good and evil, conceived through analogy with their own virtues and shortcomings. Seeking for themselves a noble heritage in the domain in which they find themselves, they will claim to have been created in the image of that God, whereas, it would really appear that they themselves have created God in their own image; and by thus attributing to Him many characteristics of their own natures, they will have sought to excuse some of their own vices by claiming them as noble attributes of the god they have created. If you talk with this group in an endeavor to understand what has happened in the birth of the cathedral, you will find so much that is extraneous to the matter in hand that any explanation which may be offered you in the form of reasons and methods will seem to call for more need of explaining the reasons and methods than for explaining the cathedral itself.

And if you go to the other group and inquire why it was that the cathedral seemed to have been anticipated in the forms inherent in the rubble, they will possibly tell you cold-bloodedly that there is nothing very much to be astonished at. They will say that naturally there were some forms in the rubble, and the cathedral happened to be the structure that came out of it. If there had been other forms, some other structure would have evolved, and they will tell you sarcastically that you probably would have been just as much impressed with that. And yet I think you will feel a little dissatisfied about the whole business.

Now in this matter of the cathedral, I have, of course, spoken in a kind of parable. What, in nature, is the real story of which this story I have told is the parable?

A thousand million years ago, our sun, like billions of its kind, stood lonely in the universe, lonely in the sense that its light, traveling at 186,000 miles a second, took years to reach its nearest neighbors. This fiery orb was our mass of rubble. Its existence would seem without obvious purpose other than that of radiating its substance in the form of light and heat to the boundaries of the universe or to unlimited space, vying in this aimless activity with all of its fellows more numerous than the grains of sand now upon our shores.

It is important to realize that none of the essential large scale elements of our existence had any place or meaning in this fiery orb. There was no life, plant or animal, nor could such have existed under the conditions then prevailing. There were no seas, there were no ships, there were no automobiles, there were no steam-engines, there were no electrical machines, no dynamos, no motors. None of the gadgetry of electronics existed, nor could it have existed in that great cauldron of fire. And yet, the astonishing thing is that the possible functioning of all of these devices and machines today is dependent upon something which had full meaning in those eons of the past. That something is the law structure inherent in atomic and nuclear physics. The elements of regularity inherent in that law structure are the elements of regularity inherent in the individual pieces of rubble in my parable, and the Notre Dame of that parable is our civilization of today.

Just as Notre Dame was anticipated in the forms of the building pieces of the rubble of my parable, so our civilization, we ourselves, the great globe on which we live and all that it contains, were, in a sense, anticipated in those laws of atomic behavior which, in the past, seemed hidden in the activities of chaos and which then had not the opportunity of exhibiting the potentialities inherent in them, for the environment in which they functioned, an environment created indeed by their own nature, would have destroyed any attempt to exhibit these potentialities.

Now, I cannot claim that the task of science is complete in providing from the atomic laws, as man at present knows them, the story of all the history of the universe. In the inanimate world we have gone far. Much remains to be done in the great problem of life and all that it implies, but even here the way is beginning to show itself in a form sufficiently well in harmony with the laws of inanimate things to encourage the belief that the pattern of life itself is dictated by relatively few ultimate principles, as dead matter appears to be governed.

Let us pause for a moment while we contemplate some of the elements of the fundamental structures—the atoms of matter—from whose potentialities our universe was born. Most of us know that every-

thing around us is composed of some hundred elements. We know also that those elements like copper, iron, hydrogen, and so forth, are composed of small structures called atoms which in turn are built out of certain fundamental particles. These particles are essentially of three types, positively charged particles called protons, negatively charged particles called electrons, and neutral particles called neutrons. At the center of the atom is a core composed entirely of neutrons and protons. Around this core, at distances which are very small, but enormous compared with the dimensions of the core or nucleus, are to be found electrons equal in number to the protons in the nucleus. These electrons perform a kind of dance around the central nucleus which acts for them as a controlling influence, dictating their motions as our sun controls the motions of the planets. The nuclei, or cores, of the atoms are so small that if everything in the universe were magnified until such a typical core appeared of the size of a pin's head, that pin's head would, on the same scale of magnification, have a diameter comparable with the diameter of the earth's orbit around the sun.

The light atoms have few protons and neutrons in their nuclei, the heavy atoms have many. Thus, hydrogen, the lightest of all elements, has only one proton in its nucleus. Helium, the next lightest, has two protons and two neutrons. One of the heaviest elements, uranium, has 92 protons and about 146 neutrons in its nucleus, and around this nucleus is a ballet of 92 electrons.

It is for the most part the characteristics of this ballet of dancing electrons which determine the characteristics of any particular atom in relation to its fellows as regards the formation of chemical compounds—compounds like water, for example, whose smallest individual representative, a molecule of water, contains two atoms of hydrogen and one of oxygen. When two atoms come near together, the members of their ballets cooperate to produce a dance more complicated than that of either of the individual ballets, a dance which locks them into one meaningful structure controlled by the two ballet masters, the nuclei of the two atoms.

Now the atomic nuclei, composed as they are of protons and neutrons, have their own individual structure, but it is a structure controlled by its own internal laws, a structure which is practically unaffected by the activities of the ballet. In other words, chemical phenomena concerned with the combinations of atoms have practically no effect on the atomic nuclei, even as the motions of the planets, controlled as they are by the sun, have very little effect upon those dramatic occurrences which we see going on in the sun itself—the terrific explosions sending flame hundreds of thousands of miles into space, and the many other things with respect to which time, in our present epoch, is enriching our knowledge continually.

It is only comparatively recently, and by the utilization of tech-

niques beyond the realm of chemistry, that we have been able to probe the nuclei of the atoms and learn something of the rich store of regularity and law form locked in them. In the process of peering into these matters, many things have revealed themselves. We have learned of the enormous stores of energy locked in the nuclei. We have learned how to release the energy, as in the atomic bomb. We have learned that extraordinary things happen when the nuclei, so quiescent in their normal state, become irritated by the treatments to which we can subject them and to which, indeed, they are being continually subjected in such phenomena as cosmic rays. Under the stimuli of such irritations, new kinds of particles can be born and play their role in the picture of what happens. Such particles, however, come into existence only in these states of turmoil, and when the nucleus is at peace or when they depart from its influence, they die rapidly and so disappear from the scene. All of these strange phenomena, and the milder phenomena of chemistry, owe their origin ultimately to the law potentialities of these neutrons and protons which lie locked in the nuclei.

But the story does not end even here for it now appears that the atoms themselves, with their nuclei, represent but an intermediate stage in the development of nature and that, indeed, there was a time when there were no atoms—when, in fact, there was nothing but a mass of neutrons—in the universe, and in the properties of those neutrons, alone, lay all the potentialities of nature, in the sense that from them were born all other things, and from their properties were born all the properties of those other things. We have reason to believe that this primordial situation existed some four thousand million or more years ago and that at that time the neutrons, the only material inhabitants of the universe, existed in the form of an extremely dense gas, in company with heat radiation, born itself of the activities of the neutrons. Truly, at that time the universe was without form and void, but we can hardly say that there was darkness on the face of the universe, for the temperature in that epoch is believed to have been of the order of ten thousand million degrees. It is true that at such high temperatures practically all of the enormous amount of radiation present existed in the form of a kind of light—very hard X-rays, in fact, or gamma rays, as we call them—to which an eye, if it could have existed, would have been insensitive as regards a recognition of what we call light; but some of the radiation, enormous in total amount although a very small fraction of the whole, would have produced, to a hypothetical eye viewing it, a brilliance far surpassing anything man has ever seen either in the sun or the atomic bomb.

Our present knowledge of atomic laws enables us to picture the subsequent history of this state of affairs and to realize that as this neutron gas cooled and expanded the law inherent in the neutrons and their progeny, the radiations, would give birth to protons, which in turn, by

participating in the story, would cooperate with the neutrons in giving birth to the atoms as we now know them.

Of course, you may well ask as to the state of affairs which preceded even that time when all the neutrons were collected as a dense gas and when there were no atoms. Alas, from here backwards the story becomes hazy, but it is not necessary to complete the story in order to understand the subsequent evolution from that period of brilliant chaos which we have envisaged as prevailing when the neutrons and the radiations were the only offspring of the universe; and, we have to realize that even at that time the potentiality of all that has happened since, the potentialities represented in the growth of structure in the universe, in the appearance of planets, and in our own appearance on the scene, with the subsequent developments of our civilization, all stem back to the laws inherent in the neutrons in that epoch of the remote past.

And now, I must leave this gigantic story which man, with the knowledge which he has gained from the present, has been able to piece together as regards the past. It is impracticable for me to cite all the details of operation of the laws of nuclear physics pertinent to these matters. It is, however, of interest to point out one or two respects in which what we may call the necessities of our universe were anticipated in the fundamental laws which existed before those necessities were called for by man's existence.

First we have the permanence of matter itself. The steel of which our ships are made does not change overnight into something else or evaporate from view. This fact is, perhaps, a matter of greater astonishment to the man of science than it is apt to be to the layman, for the best a physicist could do until quite recently in the matter of proposing laws suitable to account for the atom's behavior was to produce laws which would inevitably have resulted in the atom's immediate self-destruction. Those aspects of the modern quantum theory which are most vital in providing for the permanence of the atoms and so for matter in bulk are aspects which run most contrary to what may be termed our intuitive feelings of common sense.

The existence of radiation such as light, heat, and wireless radiation is a consequence of the fundamental laws of physics, and these things form the bases of our communication. Moreover, the photoelectric effect, which is the power possessed by light to eject electric particles from surfaces upon which it falls, and a kindred property, that of producing rearrangements of the electronic constituents of the atom when light falls upon it, constitutes a mechanism responsible for our ability to see.

In the sun, even today, the solid state is unknown, and there is no rigidity to anything, yet the strength of steel and of solid materials in general, so necessary for the carrying on of man's activities, depends

upon characteristic properties of the atoms as they exist in the sun today and have existed in the past in spite of the impossibility of their showing their potentialities in that environment. The easy flow of fluids, their power, as in the case of gases, to expand and contract, are basic to innumerable requirements of the activities of man, and not the least of these is the requirement imposed by his necessity to breathe.

There is nothing of the nature of a chemical compound on the sun. There is no water. There are no acids, there are no alkalies. There is no protoplasm. Even the atoms of the elements themselves are for the most part torn asunder. I need hardly point out the importance of chemical activities to the economy of civilization. Almost everything depends upon them. Man's structure itself and the mechanisms by which he lives are dependent upon them, and all the potentialities for the operation of the laws of chemistry are known and anticipated in the fundamental laws which prevail in the sun, where, however, those laws of chemistry have no chance whatever of exhibiting their activities.

But the fundamental laws of the universe provide, in their potentialities, not merely for the necessities of existence as we know it today, but for the conveniences and even for the esthetic enjoyments of mankind. It is very convenient for man that there should be a substance as strong as steel. It is very convenient that there should be a substance like water, which will dissolve so many things and perform the multitudinous functions which only water can perform. Perhaps it is more than convenient and is an essential that there should be a substance like chlorophyll, capable of transforming the sun's otherwise useless energy into a form which provides for plant life. It is very convenient that there should be a relatively inert gas like air which, among the many services which it performs, can be held to the earth as an atmosphere and form a means of communication through the agency of what we call sound.

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It is an added boon to mankind that these sounds can assume a form pleasant to his hearing and permit, therefore, of the realization of arts such as poetry and music. The minimum requirements of absolute necessity and convenience are enhanced by the esthetic pleasure rendered possible through the beauties of foliage and the scent of flowers. That the fundamental laws of matter should provide for the realization of all of those things which we encounter in connection with electrical engineering is astonishing indeed.

Of course, man, himself a part of the whole structure, has played a great part in developing the potentialities of nature's laws. There was little to guide him to what he achieved in building a dynamo, a motor, an automobile, or a steam-engine, and yet the fact that man can claim the credit for having evolved these things does not reduce the wonder

inherent in the existence of the possibilities of their being evolved, which possibilities go back to the fundamental laws themselves.

In contemplating the tremendous achievement involved in the formulation of laws which shall have as rich a content as have the actual laws, I have sometimes meditated upon the supposed actions of a very rabid materialist who thinks that there is nothing very remarkable in the design of the laws. I ask such a materialist himself to design a universe in the sense that I will allow him to assert any basic laws he wishes and will guarantee for him that they shall apply in the universe which he intends to build out of them. He will probably choose his laws so as readily to account for some of the facts of nature which have most intrigued him. He will probably account in a very simple manner for the spectrum of hydrogen and for the chemical properties of a few important compounds, but when all is done, I shall be surprised if it does not turn out that his laws have failed to predict a photoelectric effect, so that he will be blind. Or if he has remembered this necessity, I shall possibly find that his laws fail to provide for a substance like wood, hard enough to build houses of, but soft enough to drive nails into. I think he may have forgotten to provide for a nice substance like mercury, so convenient in research, and as a result his scientists will be greatly hampered in their researches. And so, as an end thought, I picture our very materialistic physicist who has designed his universe and then, as a hell, is condemned to live in it.

It is an interesting and peculiar fact that in a very large number of cases inanimate nature has not waited for man to develop the consequences of what is around him but, in concealed form, has frequently used principles and produced devices which, at a later date, man has himself fashioned, only to find that nature has anticipated him. Indeed, if inanimate nature could be endowed with a personality capable of making claims, many of the patent applications of human beings would have to be granted with strict reservations in favor of what nature has done.

The birds solved many of the problems of flight long before man had successfully flown. The structures of their wings and the potentialities for maneuverability are marvels of structural design.

Lens design is an art invoking the highest skill and we have good reason to be proud of the clear definition obtainable from a high grade modern camera lens. In Roy Chapman Andrews' *Nature's Way*, the author says: "Often, on the Gobi Desert, after killing and then skinning an animal, I would leave the carcass. With ten-power binoculars I would scan the heavens in every direction and then sit down some distance away to time the arrival of the black vultures, one of the largest birds in the world. Usually, in not more than fifteen or twenty minutes, I could pick out tiny specks in the sky with the field glasses. Wheeling in wide circles, the great birds would drop lower and lower, finally to

land upon the carcass. Doubtless they had been watching the skinning operation from far beyond the range of human sight."

Here is a fish for whom nature invented bifocals. He found it necessary to swim so that he could see things in the water and at the same time see things in the air above, so he has two sets of pupils to his eyes, one above the other, and he can swim so that one sees what is in the air while the other sees what is in the water.

There is a fish called *periophthalmus*, who spends his life partly in the air and partly in water, and he found it necessary to develop certain muscular adaptations to the lenses of his eyes, so that he could render them serviceable for vision in either medium.

Science is very proud of its achievements in the detection of small quantities; but, until the era of cosmic ray research in which man has learned to observe individual atomic particles, few of his achievements could compete in sensitivity with the noses of many animals, including his own nose. The moth is the champion of all smellers. During the breeding season, it can sense its female companion two miles away. The power possessed by the dog to follow a scent, is something as yet not understood.

Even the human nose is quite remarkable in its sensitivity. In this connection, I sometimes cite what I call the problem of the flapper. The flapper puts one drop of perfume on her handkerchief. When it is dried, it weighs perhaps a microgram. Now the flapper walks ten miles a day for three days, and if you put your nose anywhere within the cylinder of about a yard's radius surrounding the path of the flapper and breathe in about a cubic centimeter of air, you can smell that perfume. In other words, a microgram of perfume is spread throughout a cylinder two yards in diameter and thirty miles long. You take a cubic centimeter of air from that cylinder and your nose can detect the perfume in it. In case anyone thinks I am exaggerating a little in this matter, I will admit that I have never actually tried the experiment, but the principle remains.

One of the great achievements of the last war was the invention of radar, in which objects like airplanes reflect electromagnetic waves of short wave length emitted from a home base, back to the base. However, the principle of radar was known to the bat long before. The bat is blind but he emits squeaks and chirps as he flies. These are sounds of very high pitch and high pitch is the essential feature necessary to enable them to define with precision, by their reflection, the object which reflects them. And so, by this device, the bat detects objects in his path.

At the beginning of the last war, great use was made of dive bombing, by which principle the pilot was able to attain excessively high speeds by using the influence of gravity and yet cause the plane to rise again before striking the earth. However, long before this, the gannet

was an expert in the art of dive bombing. Perhaps he does not actually bomb, but he does get down very rapidly to capture what he is after.

The use of smoke screens is common in modern naval warfare. However, the octopus invented it. He squirts into the face of his enemy a dark liquid something like India ink which he stores in a bag concealed within his body. Behind this smoke screen he can escape when threatened.

Incidentally, the octopus could claim a patent for another modern invention, namely, jet propulsion. He can emit from a spacious cavity within his body a stream of water with high velocity. The reaction from this jet drives him backwards as fast as an ordinary fish can swim and enables him to get out of the way quickly when he so desires. His forward motion is limited to a crawling one, in which his flexible arms and suckers participate.

Modern warfare has exploited fully the potentialities of camouflage, but these potentialities have been used, from time immemorial, to a much greater perfection and in an enormous variety of ways by the animal kingdom. Everyone knows of the chameleon. Again, this picture is composed of an insect and a branched stick, but it is not very easy to tell which is which.

It is only comparatively recently that man has used invisible light, in particular, infrared, as a means of locating objects when visible light is unsuitable or undesirable. However, the rattlesnake knows of this device. If you tape over a rattlesnake's eyes and swing a warm electric bulb in front of his head, he will strike at the warm object unerringly.

The power of the homing pigeon to return to his base is still a mystery. Some evidence has been given to support the view that he possesses apparatus which enables him, like a mariner, to make use of the earth's magnetic influence.

It is strange that nature does not seem to have made very much use of electricity. Here indeed man, who is after all part of nature, can claim some *kudos*. However, there is a creature, the electric eel, who can generate electrical potential differences from four to six times as much as those in our lighting circuits, and it has been said that he is able to provide a thousand watts in electrical power, that is, enough to light 20 ordinary bulbs. I am not prepared to say how long he could act as a power plant in this matter.

All students of acoustics know of the potentialities of resonators. However, the kangeroo rat is rather an expert on the matter. He adopts two ways of hearing, one through the air and the other through the ground. For the latter purpose he has evolved large resonance chambers which take up half of his skull space and enable him to detect approaching danger through the ground vibrations which it produces.

It may not be too much stretch of the imagination to regard this

fish, the Archer fish, as using the principle of the anti-aircraft gun. He likes flying insects but cannot reach them from his habitat in the water, so he shoots them down with water flak, which he ejects straight at his quarry, causing the insect to fall into the water, wet and battered, an easy prey to his expectant mouth.

And among all of these complex mechanisms, perhaps the most marvelous are those comprised in that conglomeration of ingenious devices, electrical, mechanical, acoustical, optical and chemical, which serve the needs of the human body.

Some years ago, I had occasion to have constructed a device which could go into a glass vessel through a small opening and could then be opened up into a desired form. The head of our shop and I spent much time in meditating upon the best way of accomplishing the desired end. Finally, I said: "Let us see how Nature solved this problem." For it seemed that efficient folding is the kind of thing required in a flower. On examining the way in which the folding was brought about in a flower, it turned out that that was the best method to adopt in our particular problem and we made the device accordingly.

It is a pretty good guess to assume that if there be any new phenomenon which makes its appearance in the course of research, somewhere or other nature has used that principle, and it very frequently happens that when unconscious nature, and that part of nature which is man, have used a device, unconscious nature has used it in better form. It is only fair to man to say that sometimes this favorable condition is brought about by nature's use of facilities unavailable to the ordinary human experimenter. Thus, sometimes efficiency depends upon the smallness of the structure, or the large numbers of similar kinds of structures. Man is limited in the smallness to which he can reduce things and the numbers of them which he can make, but industrious nature is unlimited in these matters and has no trade union to curtail its activities.

Now with the picture as I have envisaged it before us, what are we to surmise as to the meaning of it all? In appraising the significance of the situation it is desirable to separate facts from dogma. We first raise the question: "Is the universe of intelligent design?" I think that the answer must be definitely in the affirmative in the sense that the kind of ingenuity which is evident in the workings of the universe is, on a large scale, the same kind of ingenuity that we might expect from an extremely clever engineer. In fact, if there had been an engineer who had devised all the various schemes which we find in nature's operations, we should certainly regard him as the greatest genius of all time, even though we did not attribute to him the responsibility for having created the basic laws of atomic physics on which everything depends. Success in utilizing those basic laws to the ends which we see around us would be sufficient to guarantee his

reputation. It must be clearly understood that in the statements which I have made, I have implied no vestige of assertion of the existence of an intelligent being in this matter. That would involve us in the endless discussion of how the being came into existence, and I must defer that question. I simply maintain that the evidence around us shows the same kind of ingenuity as that which we associate with the products of intelligence.

Now, you will observe that in introducing our imaginary engineer, to explain what I meant by intelligent design, I relieved him of the responsibility for creating the initial fundamental laws which provided the potentialities which he has developed. When I ask as to the origin of these laws themselves, I face a different kind of intelligence, an intelligence which transcends even that of that most intelligent of all engineers, for no engineer has the task of inventing his own laws. His task is simply to exploit them. This greater intelligence had to formulate these laws and to foresee, in their formulation, all of the consequences which our engineer has developed. One of the most marvelous things is the fact that the ultimate formulation should have been as simple as it is. The man of science has always admired the process of the development of rich complexity out of ultimate simplicity and, indeed, the great goal of the theoretical physicist has always been to seek the simplest form of laws which are consistent with the complexity of the consequences of those laws.

One of the greatest examples of this kind of thing is to be found in Newton's discovery of the so-called law of gravitation. Before Newton's time, the motions of the heavenly bodies were a profound mystery. Kepler succeeded in reducing the description of the planetary motions to something which was the consequence of a few empirical laws, but Newton showed that these empirical laws of Kepler were themselves but the consequence of one simple law, the law of gravitation. From that law, everything else could be deduced, the motion of the planets, the times of occurrence of eclipses and a hundred other things. The virtue of the law lay in the fact that by saying one thing, one could deduce as its consequence many things. Even though one is not able to give a reason for the one thing or even to give meaning to a question which asks the reason for it, much has been achieved in showing that the assumption of that one thing includes so many fruitful consequences. If I were facetiously-minded, I would say that, while the function of the teacher is to take a complicated thing and make it seem simple, the function of a creator is to take a simple thing and make it complicated, so that in the complication, the richness of the universe may be achieved. But even Newton's law of gravitation was only a step in a single domain. There are the laws of electricity and magnetism, and many other laws, where simplicity in the formulation of the fundamentals has been achieved in the several domains. The further

great thing is to regard this conglomeration of simple laws as itself a complexity, with the members of that complexity related as brothers or cousins, in that they themselves are born of still fewer fundamentals. The art of a creator, if there be one, is to start with the fewest fundamentals and extract from them the richest consequences. In the laws of nuclear and atomic structure, we believe we have gone far towards finding those ultimate basic laws whose consequences are all we see around us. Undoubtedly much remains. The mystery of life has not yet been well fitted into the picture; but our experiences of the past give us ground for thinking that if simplicity appears not in the correlation of the laws of animate things with those of inanimate nature, the fault lies with us in not having found it, rather than with the ultimate basis of the correlation itself.

In what I have said so far, I have emphasized what I claim to be an observed fact, namely, that the universe is of intelligent design. The intelligence is of two degrees. First we have that involved in developing the consequences of the fundamental laws. The intelligence here concerned is analogous to that of a very clever engineer and this, indeed, is my criterion involved in using the word "intelligence." But then there is a further intelligence, never required of an actual engineer, the choice of the laws themselves, so that the laws chosen shall have the richest consequences.

Now, in the light of this picture, am I to believe in the existence of a so-called intelligent being, or am I to throw everything aside and say, "I will concern myself with the facts and not worry about how they came about"? In the last analysis, I may maintain that everything came about by chance.

In order to appraise the significance of such attitudes here represented by their extremes, let me take an analogy, on a scale much more modest than that which involves the universe as a whole but which nevertheless duplicates some of the essential features.

Suppose I come into a certain country and start to observe the happenings which affect my life. I must ask you to believe that somehow or other I am unconscious of the reasons for things happening as implied in the existence of laws and individuals concerned in the government of the country. Suppose the first thing that happens to me is that I fall down and break my leg. A man in uniform comes and looks at me, and after a little while a conveyance labeled "Ambulance" arrives. It carts me off to a place labled "Hospital," where somebody X-rays the leg, binds it up in splints, puts me to bed, so that, not to trace the matter in too great detail, I am ultimately discharged as cured. Now, if I am a complete materialist in this matter, I shall regard the uniformed person who looked at me when I was on the ground as playing no function in the case. The fact that he wore a uniform was incidental and, having looked at me, he possibly passed on his way with-

out further thought. The arrival of the ambulance will be to me another accident, as will be the fact that it happened to stop just where I was. The process of picking me up and putting me into it will have no relation to anything in particular. They might just as well have picked up that other fellow who, from the equipment he carried, was evidently going to play a round of golf. I might regard the stoppage of the ambulance at the hospital as an accident, or perhaps, to put a little more structure into the matter without too much personality behind it, I might suppose that the hospital is a kind of center of force with the characteristic that it attracts ambulances and repels other vehicles. When I get into the X-ray room, I must regard myself as just a piece of material which happens to be there and gets X-rayed. I can go on in this vein until the whole history of my cure and discharge from the hospital is covered. If somebody comes to me afterwards and says: "My friend, do you know how it came about that all this happened?" I shall probably say: "No, I don't, and I don't care." If my visitor explains that there was a government in that country, started perhaps by some good king, who set up a system of laws which were designed to cover all sorts of different circumstances, and that it was a consequence of these laws that, through various complicated ramifications, the uniformed individual who saw me on the ground telephoned for an ambulance which took me to the hospital for the purpose of having me cured; if my visitor tells me all this, I may ask him for his criterion that such beings as he has envisaged in the total story actually exist. I shall ask him whether he ever saw that king of whom he spoke. He will probably tell me that the king lived many hundreds of years ago. I shall ask him whether he has any evidence that anybody else ever saw him. I shall ask him whether he has ever seen a session of what he calls the present government of the country, or even one of the trustees of the hospital, to whose activities he has attributed so much. It may be that he is unable to satisfy me on any of these points, and if he continues to try, I shall request him to pester me no further. I shall ask him in any case why he wishes me to believe all this. And now he may give me one of two types of answers. In the first type, he will say, "My friend, I will tell you why I wish you to believe all this. I think it is only right that you should go to the City Fathers and thank them and the good king, whose spirit still lives, for what has happened to you." My visitor may go a little farther and tell me that the great king and his court are apt, at times, to be very severe, and it would be well to keep in with them, for if I do not, there is a good chance, amounting in fact to dead certainty, that something terrible will happen to me some day and will go on happening for ever and ever. In olden times he might have suggested burnt offerings as a suitable form of appeasement, which ritual seems to me rather unkind to the animal which is offered. If I do not fall in with

his line of thought, my visitor may become more threatening on behalf of his deity or deities and, as a warning, he may tell me of things which his deity has done in the past and may do in the future—things which imply punishments so severe that I would never dream of inflicting them on my worst enemy, if I had any enemies. And then, in the same breath, he will insist on telling me that this deity of his imagination is infinitely good, infinitely forbearing, infinitely merciful. I will confess to becoming confused in trying to reconcile such attributes with the activities of the deity in doing unkind things which I would never do myself, although I have never regarded myself as a particularly good person.

Now, my visitor might have made quite a different appeal, when he was trying to convince me that all that had happened was not a mere accident. He might have refrained from being too specific about the details of the individuals whom he supposed to be involved and he might have concentrated on pointing out to me that if I would only recognize the purposeful element in the so-called laws which govern that country, I should be able to use those laws to the best advantage for myself and everybody else who might be concerned. If I simply believe that the ambulance found me by accident and picked me up for no reason, I shall have very little hope that if I break my leg again another ambulance will come along just in time, and I shall have little hope that if I happen to contract tuberculosis, anything will be done about it. If, however, viewing what I may call the purposeful design around me, regardless of the designer, I am willing to place some faith in the fundamentality of this design, regardless of its origin, I shall acquire a great sense of comfort in the belief that I am not alone, subject to the whims of whatever accident may befall me, but that my future, as well as my present, is somehow or other involved in the general design which seems to control the affairs of the country.

To return from my analogy of the country with its laws and government to the universe and to my place in it, I would maintain that possibly I can make greater use of the richness of any design of purposeful form which my experience shows me to exist in the universe, provided that I acknowledge to myself the reality of that existence. If I attempt to be too dogmatic and seek to attribute the design to some being, I am faced with the never-ending difficulty of accounting for the being, and I add nothing to the use which I may make of the recognition of the design.

Consider one of the lower animals, such as a dog, who yet has a finite amount of intelligence. This dog finds that there is a certain order in the house in which he lives. His food comes at certain times. He is taken care of when he is sick, and indeed, he senses a certain purpose, with his welfare at heart. He would probably understand the situation if I should tell him that he had a master who looked after his

needs. However, the situation might really be much more complicated. It might be that the dog was a ward of a society interested in the welfare of animals, or indeed, it might be that he was the ward of a less interested group of officials who operated on the laws made out by a society interested in animals, but a society which no longer existed. It may be that the society originally came into existence as a result of rather complicated considerations involving the government of the community as a whole, and things might ramify from here with greater increasing complexity, so that it would indeed be difficult, in the last analysis, to say just what person, group of persons, or other agency was responsible for the ultimate welfare of the dog. Now, I fear if I try to explain this matter to the dog, he will be greatly confused. All he really needs for the purpose of accommodating himself to the things which govern his welfare is a kind of faith in what we may call the first stage of this complexity of agencies. Indeed, this stage, represented perhaps by his master, is all that he will probably be able to comprehend. I, a human being of greater intelligence, can perhaps comprehend more. I can think of all of these agencies and governments to which I have referred, but finally, I also reach a limit which might perhaps be passed by some being more intelligent than I. However, for my purpose, as for that of the dog, what I really need to utilize fully my opportunities is the recognition of intelligent design which, if I wish, I may incorporate into the picture of a deity as its cause, and if I am a pure mathematician, I shall define this deity in terms of the sum of his attributes.

Again, if I am a mathematician, I may regard my immediate personal deity as a first approximation whose limits are reached when I ask questions as to his origin. It may be that satisfaction as regards the origin may only be understood in terms of a second approximation involving entirely new concepts in its formalization, concepts perhaps beyond my human intelligence. And then again, for an intelligence which grasps this second approximation, there might be a limit necessitating, for further comprehension, yet another formulation and a yet higher intelligence to comprehend it. And so one can imagine this process going on ad infinitum in a manner somewhat similar to less ambitious procedures familiar to the mathematician in questions of convergence of infinite sequences of processes. I think it perhaps well that I should not pursue this matter further at this time.

If I accept the principle of intelligent design, it may still be asked whether, apart from quibbles, the existence of a supreme being is not implied as a logical consequence. I have tried to draw what I deem to be the essential distinction in this matter. If, however, it may be asked why I object to go the whole way and accept such a being in the customary elementary sense, my answer is that I am concerned with the irrelevancies which accompany such an acceptance and which, as a fungus growing upon it, hide many of the beauties inherent in the con-

cept it was designed to adorn. Many such encumbrances are the products of an age when attributes now deplored were than admired. The Jehovah of the Old Testament, demanding sacrifices after the manner of the pagan gods, gives me a picture which I cannot greatly respect. Even vain man of this age would feel no great pleasure from the kind of homage paid to the gods of old and even to the God of the Old Testament. These gods were created by those who worshipped them, and autocratic authority, with fear of the consequences of disobedience, was the guiding element which controlled mankind in his relation to them. In the present age, I find the known fact of the baleful effect of high living on my digestion a more potent deterrent than the fear of hell fire if I imbibe too much. At least I know what it is that I am fearing in this matter.

I will not deny that there may be primitive ages and primitive people in which the decrees of authority founded upon imagery may serve a useful purpose where the civilization or mental development is such that something more meaningful cannot be comprehended, but I am speaking now of these matters, not in their relation to such imagery, but in their role as candidates for acceptance in absolute truth.

A personal deity has always been regarded as a being who is susceptible to influence by prayer. Now, strange to say, in spite of all the materialism which I seem to have uttered, I am one who is not averse to belief in the fact that something akin to prayer can have beneficial effects. I am apt to regard it, however, as all that we yet have for setting into motion that relationship between mind and matter as yet but little understood, but which, in the last analysis, may be more profound than many of us surmise. As to what form it takes matters, in my judgment, but little. I will confess that I am occasionally irritated by the kind of prayer in which the approach, in contrast to the expression of a strong desire, takes the form of the citation of circumstances known to everybody, and ends with what appears to be advice to the Deity. I would suppose that the form most appropriate to any individual or group of individuals was that in which his mental concentration and expression of desire for what he sought was most intense. Perhaps rather remotely related to such matters is the confessional of the Church of Rome, whose modern equivalent seems to be the ministrations of the psychiatrist, with the difference that the latter practitioner charges $25 an hour for listening to what one has to confess.

Finally I come to that aspect of religion which is concerned with guiding the way of life of mankind. In this matter, it finds, in my judgment, its highest place.

In the course of history, there have arisen great thinkers—Christ, Buddha, Mahomet, and in our own day, Gandhi. Frequently these men have founded what we call religions, but the essence of what they

have formulated consists of plans of living, calculated to promote the greater happiness of mankind. These prophets were willing to illustrate their creeds by living the lives they advocated. They suffered and sacrificed for their beliefs and thus they won the confidence and respect of vast sections of humanity, so that by many they have been deemed gods, but gods whose attributes were those of compassion and kindness and whose doctrines were rarely enforced by acts of brutality, however much they may have been distorted into this form by later perversions of the original kindly ideal. To picture some of these great beings as divine may, in many cases, have served a beneficial end, and indeed may so serve today.

Now it is quite true that the complexities of the situations encountered in our lives are such that if any one of these beings were asked to provide written answers to all of the problems which could be conceived as arising, he would probably have to write more books than have been written by all the lawyers of all time for the confusion of mankind and the perpetuation of their species. Nevertheless, a comparatively few utterances, as in the "Sermon on the Mount," serve to define with reasonable clarity the spirit of a belief for a range of circumstances far wider than those dealt with in the utterances themselves. Some simple old lady, who had spent the whole of her eighty summers in primitive surroundings, would be greatly confused if, in answer to one of her problems, she were presented with the numerous treatises of the lawyers above cited. However, by forming the concept of a being who would act according to the principles of the "Sermon on the Mount," for example, she defines for herself a picture; and the subconscious logic in this picture enables the being of her imagination to give, in response to her question, an answer which is likely to be good. Whether or not our old lady chooses to think of this being as a god, I care not. If, in her search for an answer to her problem, it helps her to visualize this god in some specific form—as a man with certain features, for example—I have no criticism. As a man of science, I do the same thing daily in drawing pictures of the atoms for the purpose of stimulating my brain to think about the problem in hand, and I do this even though I may know that the picture is, in the last analysis, unnecessary. And so, if you ask me for the meaning of a religion, I am constrained to reply that it is a code of ethics draped about the image of a being as an ideal.

In returning once more to the trend of events in the universe regarded as a growing structure of intelligent design, there is one matter which looms in importance above all others. It concerns the question as to whether the trend is continually towards something which gets better as the ages advance; better, I mean, in the sense that if life continues, either in visible form, as we view those who live today, or in invisible form as theology has viewed the hereafter; whether things get

better in the sense that that which lives in one form or another shall get happier and happier, or whether misery, boredom, or ultimate extinction is the doom of humanity. If ever faith were called for, it is in this matter that it must be invoked. The various religions have adopted a rather simple solution. There are forces of good, symbolized by a god, and there are forces of evil, symbolized by the devil. The god rules heaven and the devil rules hell. Mankind, with a postulated free will, has it in his own power to decide whether his hereafter shall be one of permanent happiness or permanent misery. I will confess that this picture is not very satisfactory to me at any rate. In any case, it is born of dogma and not of an examination of the trends of nature herself. If I look once more at nature, I see as a fact a growth from a state of chaos, in which there seems to be no potentiality of happiness or of unhappiness, I see a growth apparently aimed with a purpose to provide eventually for beings who do have such potentialities. I see a marvelous provision for the activities of such beings and I see a continual improvement in those potentialities which make for happiness over those which make for despair. Truly, all is not happiness in the world today. If I am to think of a deity, I prefer to think of one whose powers, though great, are limited, and that he is doing the best he can, growing, perhaps, in stature as the universe grows, with naught but happiness as the ultimate goal. It seems reasonable to suppose that man has a greater potentiality for joy than has a snail; and as I view man and his kind, of which picture I have but the beginning—since a few thousand years is nothing in the scale of time of the universe—as I view his development over this period, I seem to see things opening up in such a way that happiness increases and misery gets less. This trend, born of direct observation rather than of dogma, is all that I have, but such as it is, to this must I pin my faith, in the hope that in the last analysis all is well with the universe. Perhaps the universe itself has its own problems in its development, so that the flowers cannot grow entirely without the weeds, the good cannot develop unaccompanied by a little evil, but if my faith can encourage the hope that ultimate good is the aim, with complete elimination of evil, then have I a basis of contentment, not merely for the present, but for the eons that are to come.

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A CONTRIBUTION TO THE THEORY OF NONLINEAR SYSTEMS

BY

L. A. ZADEH[1]

ABSTRACT

A system of classification for nonlinear two-poles is introduced and some of the basic properties of two-poles of various classes are established. The classes are designated as $\mathfrak{N}_1, \mathfrak{N}_2, \mathfrak{N}_3, \cdots$ and are such that each class in the sequence is a subclass of all the classes following it. Furthermore, the class of linear two-poles is a subclass of every class in the sequence.

It is shown that a general nonlinear two-pole of class $\mathfrak{N}_1$ is completely characterized by its responses to a family of step functions with amplitudes ranging over all real values. In an analogous manner, the dual of a general two-pole of class $\mathfrak{N}_1$ is completely characterized by its responses to a family of complex exponentials whose amplitudes and frequencies range over all real values.

Several possible modes of realization of two-poles of class $\mathfrak{N}_1$ are indicated. One canonical form consists of a parallel combination of pairs of linear two-poles and zero-memory (non-inertial) nonlinear two-poles. For this canonical form an explicit expression is developed for the response of an initially excited two-pole of class $\mathfrak{N}_1$ to a specified input.

A canonical realization for two-poles of class $\mathfrak{N}_n$ in the form of an n-dimensional delay-line filter is indicated. It is shown that certain two-poles of class $\mathfrak{N}_{n/1}$, which is a subclass of $\mathfrak{N}_n$, are completely characterized by their responses to n-tuplets of rectangular pulses and are additive with respect to such n-tuplets.

Finally, an extension of Wiener's theory of prediction and filtering to filters of class $\mathfrak{N}_1$ is briefly outlined.

INTRODUCTION

The theory of nonlinear systems has made some progress recently toward the formulation of general theorems bearing on both the internal and external behavior of broad classes of nonlinear systems (1–5).[2] The aim of the present paper is to make a further contribution in this direction through a study of a certain system of classes of nonlinear two-poles.[3] Specifically, we consider a sequence of classes, designated as $\mathfrak{N}_1, \mathfrak{N}_2, \mathfrak{N}_3, \cdots$, such that each class in the sequence includes all the preceding classes and, furthermore, the class of linear two-poles is a subclass of every class in the sequence. In other words, the system of classes $\mathfrak{N}_1, \mathfrak{N}_2, \mathfrak{N}_3, \cdots$ forms a hierarchy of families of nonlinear two-poles, with each family including the class of linear two-poles as one of its members.

As a preliminary to a more explicit characterization of the classes

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] By a "two-pole" is meant a nonlinear system which has only one input and one output terminal. Systems with more than two terminals are considered in a note entitled "Nonlinear Multipoles," to be published in *Proc. Nat. Acad. Sci.*

$\mathfrak{N}_1, \mathfrak{N}_2, \mathfrak{N}_3, \cdots$, it will be helpful to introduce a few basic symbols and define several terms. The symbols are:

N = a general nonlinear two-pole.
$N_{(n)}$ = a general nonlinear two-pole of class $\mathfrak{N}_n$.
$u(t)$ = input to N; where no confusion is likely to arise, $u(t)$ will be denoted by u.
$v(t)$ = output of N, that is, the response of N to $u(t)$. Wherever possible, $v(t)$ will be abbreviated to v.

In what follows, the symbolic equation

$$v = Nu \tag{1}$$

signifies that $v(t)$ is the response of N at rest to $u(t)$. The symbolic equation $N = N_1 + N_2$ means that N consists of the parallel combination of N_1 and N_2, as shown in Fig. 1(a). Similarly, $N = N_2N_1$ means

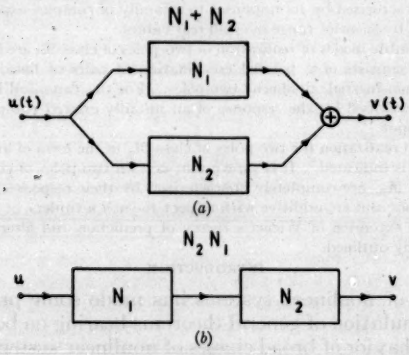

FIG. 1. Functional notation for (a) parallel and (b) tandem connection of N_1 and N_2.

that N consists of the tandem combination of N_1 and N_2, with N_1 preceding N_2 (Fig. 1(b)); and $N = \alpha N_1$ means that N consists of a two-pole N_1 followed by a device which multiplies the output of N_1 by the factor α.

The totality of all the inputs on which a specific two-pole, N, may operate will be referred to as the *input space* of N. Similarly, the totality of all the outputs will be referred to as the *output space*. In mathematical terminology, the input and output spaces correspond, respectively, to the *domain* and *range* of an operator.

A two-pole N is said to be *additive* if for any two time-functions $u_1(t)$ and $u_2(t)$ in the input space of N, the following relation holds:

$$N(u_1 + u_2) = Nu_1 + Nu_2. \tag{2}$$

In other words, N is an additive two-pole if the principle of superposition is applicable to it.

A two-pole N is said to be *homogeneous* if for any time-function, $u(t)$, in the input space of N, and any real constant α, one has

$$N\alpha u = \alpha N u. \tag{3}$$

It should be noted that (3) does not imply (2); nor does (2) imply (3). However, it is well known and can readily be shown that (2) implies (3) for all rational α.

A two-pole that is both additive and homogeneous is said to be *linear*. A *nonlinear* two-pole is one that is either non-additive or non-homogeneous or both.

CLASSIFICATION OF NONLINEAR TWO-POLES

We are now in a position to characterize in an explicit manner the classes $\mathfrak{N}_1, \mathfrak{N}_2, \mathfrak{N}_3, \cdots$, and formulate some of the basic properties of these classes.

We define the class $\mathfrak{N}_1$ as the collection of all two-poles whose input-output relationship can be expressed in the form

$$v(t) = \int_0^\infty \mathrm{K}[u(t-\tau), \tau]\, d\tau, \tag{4}$$

where τ is the variable of integration, $u(t)$ is the input and $\mathrm{K}[u(t-\tau), \tau]$ is any real function[4] of $u(t-\tau)$ and τ. Since this function completely characterizes the external behavior of the two-pole, it will be referred to as the *characteristic function*. It might be remarked that the characteristic function of a linear two-pole, as defined in a previous paper (6), is not a special case of the characteristic function defined above, although the two concepts are closely related.

It is readily verified that the class of linear two-poles is a subclass of $\mathfrak{N}_1$. Thus, the input-output relationship for a linear two-pole reads

$$v(t) = \int_0^\infty W(\tau) u(t-\tau)\, d\tau, \tag{5}$$

where $W(t)$ is the impulsive response of the two-pole, that is, the response to the unit impulse (delta-function) $\delta(t)$. Clearly (5) is a special case of (4), with the characteristic function having the form

$$\mathrm{K}[u(t-\tau), \tau] = W(\tau) u(t-\tau). \tag{6}$$

Consequently, any linear two-pole is a member of the class $\mathfrak{N}_1$.

In an analogous manner, the class $\mathfrak{N}_2$ is defined as the collection of all two-poles whose input-output relationship can be expressed in the form

$$v(t) = \int_0^\infty \int_0^\infty \mathrm{K}[u(t-\tau_1), u(t-\tau_2), \tau_1, \tau_2]\, d\tau_1\, d\tau_2, \tag{7}$$

[4] It is understood that the characteristic function may include delta-functions of various orders.

where τ_1 and τ_2 are the variables of integration. The characteristic function in this case is $K[u(t-\tau_1), u(t-\tau_2), \tau_1, \tau_2]$.

It is evident that the class $\mathfrak{N}_2$ is much broader than $\mathfrak{N}_1$. Indeed, it is a simple matter to demonstrate that the latter is a subclass of $\mathfrak{N}_2$. For this purpose it is sufficient to write (4) in the form

$$v(t) = \int_0^\infty \int_0^\infty K[u(t-\tau_1), \tau_1]\delta(\tau_1 - \tau)\, d\tau_1\, d\tau, \tag{8}$$

where $\delta(\tau_1 - \tau)$ is a delta-function. Since (8) is a special case of (7), any two-pole of class $\mathfrak{N}_1$ is also a member of the class $\mathfrak{N}_2$.

More generally, the class $\mathfrak{N}_n$ consists of all two-poles for which the input-output relationship may be expressed in the form of an n-fold integral

$$v(t) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty K[u(t-\tau_1), u(t-\tau_2), \cdots, u(t-\tau_n), \tau_1, \tau_2, \cdots, \tau_n]\, d\tau_1\, d\tau_2 \cdots d\tau_n, \tag{9}$$

where the τ's are the variables of integration, and the characteristic function, K, is any real function of the variables $u(t-\tau_1)$, $u(t-\tau_2)$, $\cdots$, $u(t-\tau_n)$ and $\tau_1, \tau_2, \cdots, \tau_n$.

It is pertinent to inquire at this point whether the system of classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$ is exhaustive, that is, whether any nonlinear two-pole can be classified as a member of one of the classes in the system. It appears that the answer to this question is that any nonlinear two-pole is either a member of the system of classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$ or can be approximated as closely as desired by a member of this system. However, it is not easy either to prove or disprove this assertion since, on the one hand, it is extremely difficult to give an explicit characterization of the most general nonlinear two-pole and, on the other, it would be equally difficult to prove that a particular nonlinear two-pole cannot be approximated as closely as desired by a nonlinear two-pole of class $\mathfrak{N}_n$, if no limit is placed on the magnitude of n.

To illustrate the classification of various types of nonlinear two-poles in terms of the classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$, we shall consider a few basic types which are of frequent occurrence in practice. In the sequel, by the statement "N is of class $\mathfrak{N}_n$" we shall mean that $\mathfrak{N}_n$ is the lowest class of which N is a member; and by the statement "N is of class $\mathfrak{N}_\infty$" we shall mean that N is either of class $\mathfrak{N}_\infty$ or might be approximated by a member of the system of classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$.

The classes to which various basic types of two-poles belong are identified below.

(*a*) Any linear two-pole is of class $\mathfrak{N}_1$.

(*b*) Any zero-memory (non-inertial) nonlinear two-pole is of class $\mathfrak{N}_1$. In this type of two-pole the output and input are related by the

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equation

$$v(t) = f[u(t)] \tag{10}$$

or more generally

$$v(t) = f[u(t - \mathrm{T})], \tag{11}$$

where f is a real function and T is a constant. To demonstrate that the zero-memory two-pole is of class $\mathfrak{N}_1$, it is sufficient to write (11) in the form

$$v(t) = \int_0^\infty f[u(t - \tau)]\delta(\tau - \mathrm{T})\, d\tau \tag{12}$$

which places in evidence that (11) is a special case of (4).

(*c*) Any nonlinear two-pole which consists of a zero-memory nonlinear two-pole followed by a linear two-pole is of class $\mathfrak{N}_1$.

(*d*) A nonlinear two-pole which consists of a linear two-pole followed by a zero-memory nonlinear two-pole is, in general, of class $\mathfrak{N}_\infty$. If the function f which characterizes the zero-memory two-pole [*cf.* (10)] is a polynomial of nth degree in $u(t)$, then the nonlinear two-pole in question is of class $\mathfrak{N}_n$. For example, if for the zero-memory two-pole

$$v(t) = [u(t)]^2 \tag{13}$$

and for the linear two-pole

$$v(t) = \int_0^\infty W(\tau)u(t - \tau)\, d\tau, \tag{14}$$

then for the tandem combination of these two-poles

$$v(t) = \int_0^\infty \int_0^\infty W(\tau_1)W(\tau_2)u(t - \tau_1)u(t - \tau_2)\, d\tau_1\, d\tau_2, \tag{15}$$

which is a special case of (7). This establishes that any linear two-pole followed by a squaring device is of class $\mathfrak{N}_2$.

(*e*) A nonlinear two-pole which is characterized by an input-output relationship of the form

$$v(t) = f[u(t), u(t - \mathrm{T}), \cdots, u(t - (n - 1)\mathrm{T})] \tag{16}$$

is of class $\mathfrak{N}_n$. This follows by writing (16) as

$$v(t) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty f[u(t - \tau_1), u(t - \tau_2), \cdots, u(t - \tau_n)] \times \delta(\tau_1)\delta(\tau_2 - \mathrm{T}) \cdots \delta(\tau_n - (n - 1)\mathrm{T})\, d\tau_1\, d\tau_2 \cdots d\tau_n \tag{17}$$

which is a special case of (9).

(*f*) A nonlinear two-pole whose output is a function of the input, $u(t)$, and delayed output, $v(t - \mathrm{T})$,

$$v(t) = f[u(t), v(t - \mathrm{T})], \tag{18}$$

is of class $\mathfrak{N}_\infty$. For, by iteration of (18), $v(t)$ may be expressed as

$$v(t) = f[u(t), f[u(t - \mathrm{T}), f[u(t - 2\mathrm{T}), \cdots]. \tag{19}$$

This relation is of the same form as (16) with $n = \infty$. Consequently, the two-pole in question is of class $\mathfrak{N}_\infty$.

(*g*) Any nonlinear two-pole of the form $N = \alpha N_{(n)}$, where α is a constant and $N_{(n)}$ is a two-pole of class $\mathfrak{N}_n$, is likewise of class $\mathfrak{N}_n$.

(*h*) Any nonlinear two-pole of the form $N = N_{1(n)} + N_{2(n)}$ [parallel combination of $N_{1(n)}$ and $N_{2(n)}$], in which $N_{1(n)}$ and $N_{2(n)}$ are two-poles of class $\mathfrak{N}_n$, is likewise of class $\mathfrak{N}_n$.

Statements (*g*) and (*h*) are immediate consequences of (9). Taken together, (*g*) and (*h*) imply that $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$ are *linear* classes in the sense that if $N_{1(n)}$ and $N_{2(n)}$ are any two two-poles in class $\mathfrak{N}_n$, and α_1 and α_2 are arbitrary real constants, then any two-pole of the form

$$N = \alpha_1 N_{1(n)} + \alpha_2 N_{2(n)} \tag{20}$$

also is a member of the class $\mathfrak{N}_n$.

Since in many practical cases a nonlinear two-pole is made up of a tandem connection of two or more two-poles, it is worth while to assign special symbols to classes of two-poles of this kind. Thus, the class of two-poles which consist of the tandem connection of m two-poles of class $\mathfrak{N}_n$ will be denoted by $\mathfrak{N}_n{}^m$. More generally, the class of two-poles which consist of a two-pole of class $\mathfrak{N}_l$ followed by a two-pole of class $\mathfrak{N}_m$ which in turn is followed by a two-pole of class $\mathfrak{N}_n$, etc., will be denoted by the composite symbol $\cdots \mathfrak{N}_n\mathfrak{N}_m\mathfrak{N}_l$. In terms of this notation, a two-pole consisting of a linear two-pole followed by a zero-memory nonlinear two-pole is of class $\mathfrak{N}_1{}^2$. Similarly, a two-pole consisting of a differentiator (which is a linear two-pole and hence of class $\mathfrak{N}_1$) followed by a two-pole of class $\mathfrak{N}_2$ for which

$$v(t) = f[u(t), u(t - \mathrm{T})], \tag{21}$$

is of class $\mathfrak{N}_2\mathfrak{N}_1$. The input-output relationship for the combination two-pole is

$$v(t) = f\left[\frac{du(t)}{dt}, \frac{du(t - \mathrm{T})}{dt}\right]. \tag{22}$$

It should be noted that in all of the preceding discussion it was tacitly assumed that the two-poles under consideration are of the time-invariant type. However, the only modification needed to extend the

analysis to time-variant two-poles is the addition of the variable t to the arguments of the characteristic function. For example, the input-output relationship for a time-invariant two-pole of class $\mathfrak{N}_1$ is given by (4), while that for a time-variant two-pole of class $\mathfrak{N}_1$ is given by

$$v(t) = \int_0^\infty \mathrm{K}[u(t-\tau), \tau, t]\, d\tau. \tag{23}$$

It should also be noted that if a two-pole has a finite memory, that is, if it operates on a finite past of the input, say from t to $t - \mathrm{T}$, then the range of integration in the input-output relationship may be restricted to the interval $(0, \mathrm{T})$. Thus, the input-output relationship for a finite-memory two-pole of class $\mathfrak{N}_1$ may be written as

$$v(t) = \int_0^\mathrm{T} \mathrm{K}[u(t-\tau), \tau]\, d\tau. \tag{24}$$

This input-output relationship can also be written in the usual form (4) if it is understood that the characteristic function is zero for $\tau > \mathrm{T}$.

Returning to the classification of nonlinear two-poles, one may raise the question of whether there exist other systems of classes of nonlinear two-poles with properties analogous to those of the system $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$. It is readily seen that a large (theoretically infinite) number of systems of classes analogous to $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$ can be obtained from this system through a process of linear transformation. Although the systems derived by this process are, in general, of restricted usefulness, there is one system, designated as $\mathfrak{N}_1', \mathfrak{N}_2', \cdots$, which is of some interest because it is the dual—in the sense of the duality between the time and frequency—of the $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$ system. The member classes of this system are formally defined by the following input-output relationships:

$\mathfrak{N}_1'$:
$$V(\omega) = \int_{-\infty}^{\infty} \mathrm{K}[U(\omega - \lambda), \lambda]\, d\lambda \tag{25}$$

$\mathfrak{N}_2'$:
$$V(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{K}[U(\omega - \lambda_1), U(\omega - \lambda_2), \lambda_1, \lambda_2]\, d\lambda_1\, d\lambda_2 \tag{26}$$

and

$\mathfrak{N}_n'$:
$$V(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \mathrm{K}[U(\omega - \lambda_1), U(\omega - \lambda_2), \cdots, U(\omega - \lambda_n), \lambda_1, \lambda_2, \cdots, \lambda_n]\, d\lambda_1\, d\lambda_2 \cdots d\lambda_n \tag{27}$$

where $U(\omega)$ and $V(\omega)$ are the Fourier transforms of $u(t)$ and $v(t)$, respectively; $\lambda, \lambda_1, \lambda_2, \cdots, \lambda_n$ are the variables of integration; and the function K plays the role of the characteristic function. The properties of these classes are essentially the duals of the corresponding properties of the classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$.

In a similar and more general manner, one may construct a system of classes based on the concept of λ-domain (6), which involves the resolution of time-functions into a set of component time-functions, $\{k(t;\lambda)\}$, where λ is, in general, a complex parameter taking values on a contour C in the λ-plane. Thus, on resolving the input, $u(t)$, into a set of component functions, $\{k(t;\lambda)\}$, we have

$$u(t) = \int_C k(t;\lambda)\,U(\lambda)\,d\lambda, \tag{28}$$

where the function $U(\lambda)$ is called the spectral function of $u(t)$. To every such mode of resolution corresponds a system of classes of two-poles which might be designated as $\mathfrak{L}_1, \mathfrak{L}_2, \cdots$, and which are formally defined by the following input-output relationships:

$$\mathfrak{L}_1: \qquad V(\lambda) = \int_C \mathrm{K}[U(\lambda-\xi), \xi]\,d\xi \tag{29}$$

$$\mathfrak{L}_2: \qquad V(\lambda) = \int_C\int_C \mathrm{K}[U(\lambda-\xi_1), U(\lambda-\xi_2), \xi_1, \xi_2]\,d\xi_1\,d\xi_2 \tag{30}$$

and

$$\mathfrak{L}_n: \quad V(\lambda) = \int_C\int_C\cdots\int_C \mathrm{K}[U(\lambda-\xi_1), U(\lambda-\xi_2), \cdots, U(\lambda-\xi_n), \xi_1, \xi_2, \cdots, \xi_n]\,d\xi_1\,d\xi_2\cdots d\xi_n \tag{31}$$

where the ξ's are complex variables; C is a contour in the complex plane, not necessarily the same as the contour involved in (28); and the function K plays the role of the characteristic function.

It will be noted that two-poles belonging to the classes $\mathfrak{N}_1', \mathfrak{N}_2', \cdots$ and $\mathfrak{L}_1, \mathfrak{L}_2, \cdots$ are not, in general, of the time-invariant type. It is not difficult, however, to construct a system of classes which include only time-invariant two-poles. One such system of classes, designated as $\mathfrak{F}_1, \mathfrak{F}_2, \cdots$, consists of the frequency-domain analogs of the classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$. The input-output relationships for these classes read:

$$\mathfrak{F}_1: \qquad v(t) = \int_C \mathrm{K}[U(s)e^{st}, s]\,ds \tag{32}$$

$$\mathfrak{F}_2: \qquad v(t) = \int_C\int_C \mathrm{K}[U(s_1)e^{s_1t}, U(s_2)e^{s_2t}, s_1, s_2]\,ds_1\,ds_2 \tag{33}$$

and

$$\mathfrak{F}_n: \quad v(t) = \int_C\int_C\cdots\int_C \mathrm{K}[U(s_1)e^{s_1t}, U(s_2)e^{s_2t}, \cdots, U(s_n)e^{s_nt}, s_1, s_2, \cdots, s_n]\,ds_1\,ds_2\cdots ds_n \tag{34}$$

where $U(s)$ is the Laplace transform of the input, $u(t)$; $s_1, s_2, \cdots, s_n$ are complex frequencies; C is a contour in the complex plane; and the function K plays the role of the characteristic function.

In the sequel, we shall be concerned primarily with the establishment of some of the basic properties of nonlinear two-poles of class $\mathfrak{N}_1$. Once these properties have been established, it is not difficult to generalize them to two-poles of classes $\mathfrak{N}_2, \mathfrak{N}_3, \cdots$, and $\mathfrak{N}_n$.

TWO-POLES OF CLASS $\mathfrak{N}_1$

In the hierarchy of nonlinear two-poles, the nonlinear two-poles of class $\mathfrak{N}_1$ are just one level above the linear two-poles. As a consequence, some of the techniques used in the analysis and synthesis of linear two-poles can be extended to two-poles of class $\mathfrak{N}_1$.

As an illustration of this statement, it will be shown first that a nonlinear two-pole of class $\mathfrak{N}_1$ can be characterized in terms of its responses to step functions or narrow pulses, in much the same manner as a linear two-pole.

It will be recalled that the input-output relationship for a linear two-pole is

$$v(t) = \int_0^\infty W(\tau)u(t - \tau)\, d\tau, \tag{35}$$

where $W(t)$ is the response of the two-pole to the unit impulse $\delta(t)$. It is readily seen that the impulsive response, $W(t)$, may also be regarded as the derivative of the response of the two-pole to the unit step function $1(t)$.

Consider now the response of a two-pole $N_{(1)}$ of class $\mathfrak{N}_1$, to a step function of amplitude α, that is, to $u(t) = \alpha 1(t)$. If we assume that the output of the two-pole is zero in the absence of an input, that is, $\mathrm{K}[u(t - \tau), \tau] = 0$ when $u(t - \tau) = 0$, and substitute $u(t) = \alpha 1(t)$ in (4), the expression for the response to the step function becomes

$$v(t) = \int_0^t \mathrm{K}[\alpha, \tau]\, d\tau. \tag{36}$$

On differentiating $v(t)$ with respect to t, we obtain the following result:

$$\mathrm{K}[\alpha, t] = \text{derivative of the response of the two-pole to the step function } \alpha 1(t).$$

This result shows that: (*a*) the characteristic function of $N_{(1)}$ is related to the step response of $N_{(1)}$ in exactly the same manner as the impulsive response of a linear two-pole is related to its step response; and (*b*) *a two-pole of class $\mathfrak{N}_1$ is completely characterized by its responses to a family of step functions, $\{\alpha 1(t)\}$, whose amplitudes, $\{\alpha\}$, range over all real values.*

Next, consider the response of $N_{(1)}$ to a narrow rectangular pulse of amplitude α and width h, denoted by $\alpha\pi_h(t)$. On substituting $\alpha\pi_h(t)$

for $u(t)$ in (4), we have

$$v(t) = \int_{t-h}^{t} \mathrm{K}[\alpha\pi_h(t-\tau), \tau]\, d\tau, \tag{37}$$

and if h is such that $\mathrm{K}[\alpha\pi_h(t-\tau), \tau]$ does not vary appreciably over the interval $(t-h, t)$, (37) reduces to

$$v(t) = h\mathrm{K}[\alpha, t] \tag{38}$$

from which it follows that

$$\mathrm{K}[\alpha, t] = \frac{1}{h} \times (\text{response of } N_{(1)} \text{ to a rectangular pulse of amplitude } \alpha \text{ and width } h)$$

or symbolically

$$\mathrm{K}[\alpha, t] = \frac{1}{h} N_{(1)}\alpha\pi_h(t). \tag{39}$$

Thus, *a two-pole of class $\mathfrak{N}_1$ is completely characterized by its responses to a family of narrow rectangular pulses, $\{\alpha\pi_h(t)\}$, whose amplitudes, $\{\alpha\}$, range over all real values.*

It is of interest to note that by dualizing this result we arrive at an analogous result for two-poles of class $\mathfrak{N}_1'$. Specifically, we have:

A two-pole of class $\mathfrak{N}_1'$ is completely characterized by its responses to a family of complex exponential inputs,[5] $\{\alpha e^{j\omega t}\}$, whose amplitudes, $\{\alpha\}$, and frequencies, $\{\omega\}$, range over all real values. It is readily seen that the same applies to two-poles of class $\mathfrak{F}_1$ [*cf.* (32)].

An important property of two-poles of class $\mathfrak{N}_1$ concerns their response to the sum of two or more time-functions which do not overlap in the time-domain. Thus, let $u_1(t)$ and $u_2(t)$ be two time-functions such that $u_1(t)$ is zero whenever $u_2(t)$ is different from zero and vice-versa. If the input to a two-pole, $N_{(1)}$, of class $\mathfrak{N}_1$ is of the form

$$u(t) = u_1(t) + u_2(t), \tag{40}$$

then by (4) the response to $u(t)$ is

$$v(t) = \int_0^\infty \mathrm{K}[u_1(t-\tau), \tau]\, d\tau + \int_0^\infty \mathrm{K}[u_2(t-\tau), \tau]\, d\tau, \tag{41}$$

since $u_1(t)$ and $u_2(t)$ do not overlap in time. The response, $v(t)$, is immediately recognized as the sum of the responses of $N_{(1)}$ to $u_1(t)$ and $u_2(t)$ when each is applied separately. Consequently, we may conclude that *the response of $N_{(1)}$ to the sum of time-functions that do not overlap in time is the sum of the responses to each time-function applied separately.* In other words, two-poles of class $\mathfrak{N}_1$ are additive with respect to non-

[5] Strictly speaking, the inputs are narrow wave packets rather than pure exponentials.

overlapping time-functions, or, in symbols,

$$N_{(1)}[u_1 + u_2] = N_{(1)}u_1 + N_{(1)}u_2 \quad \text{if} \quad u_1(t)u_2(t) \equiv 0 \tag{42}$$

where $N_{(1)}$ is a general two-pole of class $\mathfrak{N}_1$.

Again, on dualizing this statement we obtain the following result for two-poles of class $\mathfrak{N}_1'$: *A two-pole of class $\mathfrak{N}_1'$ is additive with respect to time-functions whose spectra do not overlap in the frequency domain.* In symbols,

$$N_{(1)}'[u_1 + u_2] = N_{(1)}'u_1 + N_{(1)}'u_2 \quad \text{if} \quad U_1(\omega)U_2(\omega) \equiv 0 \tag{43}$$

where $N_{(1)}'$ denotes a general two-pole of class $\mathfrak{N}_1'$, and $U_1(\omega)$ and $U_2(\omega)$ are the Fourier transforms of $u_1(t)$ and $u_2(t)$, respectively. It is readily seen that the same property is possessed by two-poles of class $\mathfrak{F}_1$.

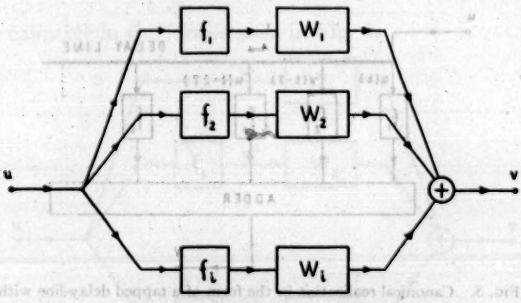

FIG. 2. Canonical realization of two-pole of class $\mathfrak{N}_1$.

REALIZATION OF TWO-POLES OF CLASS $\mathfrak{N}_1$

One of the important features of the system of classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$ is that once the input-output relationship for a two-pole $N_{(n)}$ has been specified in the form (9), $N_{(n)}$ can be realized approximately in one of a number of canonical forms. Several such forms for two-poles of class $\mathfrak{N}_1$ are developed in the sequel.

The canonical forms in question may be obtained by expanding the characteristic function $\mathrm{K}[u(t-\tau), \tau]$ into a uniformly convergent series of the form

$$\mathrm{K}[u(t-\tau), \tau] = \sum_i W_i(\tau) f_i[u(t-\tau)], \quad i = 1, 2, \cdots. \tag{44}$$

On substituting this series in (4) and inverting the order of summation and integration, the input-output relationship (4) becomes

$$v(t) = \sum_i \int_0^\infty W_i(\tau) f_i[u(t-\tau)]\, d\tau, \quad i = 1, 2, \cdots. \tag{45}$$

Strictly speaking, the class of two-poles for which the input-output relationship can be expressed in this form constitutes a subclass of $\mathfrak{N}_1$. This subclass is, in a sense, dense in $\mathfrak{N}_1$, that is, any two-pole of class $\mathfrak{N}_1$ is either a member of this subclass or might be approximated as closely as desired by a member of this subclass.

The form of (45) suggests an obvious realization which is illustrated in Fig. 2. The structure consists of the parallel combination of pairs of tandemed two-poles, the ith pair being made up of a zero-memory nonlinear two-pole for which the input-output relationship is

$$v(t) = f_i[u(t)], \tag{46}$$

and a linear two-pole whose impulsive response is given by $W_i(t)$. Thus any nonlinear two-pole of class $\mathfrak{N}_1$ can be realized approximately in the form shown in Fig. 2.

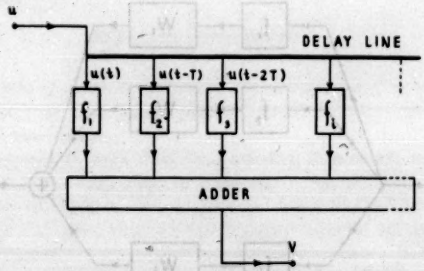

FIG. 3. Canonical realization in the form of a tapped delay-line with nonlinear weighting functions.

It is of interest to consider several particular forms of the input-output relationship (45) and the realizations associated with them. One useful special case of (45), which is particularly effective[6] when the input space consists of band-limited signals with upper frequency f_0 which is not too high, is obtained with the $W_i(t)$ functions of the form

$$W_i(t) = \delta(t - (i - 1)\mathrm{T}), \quad i = 1, 2, \cdots \tag{47}$$

where T is a constant equal to the Nyquist interval, $\mathrm{T} = \dfrac{1}{2f_0}$. With these functions, (45) reduces to

$$v(t) = \sum_i f_i[u(t - (i - 1)\mathrm{T})], \quad i = 1, 2, \cdots \tag{48}$$

which is realizable in the form of a tapped delay-line filter with non-

[6] It should be noted, however, that a band-limited signal is not completely defined over the interval $(-\infty, t)$ by its values at the sampling instants within this interval. In general, knowledge of the signal at the sampling instants between $t = -\infty$ to $t = \infty$ is required for this purpose.

linear weighting functions (Fig. 3). In this structure, the boxes labeled $f_1, f_2, \cdots$ represent zero-memory nonlinear two-poles characterized by input-output relationships of the form

$$v(t) = f_i[u(t)], \qquad i = 1, 2, \cdots. \quad (49)$$

It is of interest to note that filters of class $\mathfrak{F}_1$ [*cf.* (32)] can be realized in an analogous manner, with a bank of narrow-band filters replacing the tapped delay-line in the structure shown in Fig. 3.

Another useful form of (45) is obtained with the f_i functions of the form

$$f_i[u(t-\tau)] = [u(t-\tau)]^i, \quad i = 1, 2, \cdots. \quad (50)$$

With these functions (45) reduces to

$$v(t) = \sum_i \int_0^\infty W_i(\tau)[u(t-\tau)]^i d\tau, \quad i = 1, 2, \cdots \quad (51)$$

which is realizable in the form shown in Fig. 4.

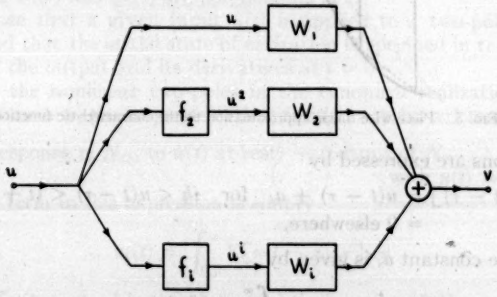

FIG. 4. A canonical realization of two-pole of class $\mathfrak{N}_1$.

A third canonical form results from approximating the characteristic function $K[u(t-\tau), \tau]$ in the piece-wise linear manner illustrated in Fig. 5. Straightforward calculation yields

$$K[u(t-\tau), \tau] \cong \sum_\nu [W_\nu(\tau)u(t-\tau) + A_\nu(\tau)], \qquad \nu = 0, \pm 1, \pm 2, \cdots \quad (52)$$

where

$$W_\nu(\tau) = \frac{1}{h}\{K[(\nu+1)h, \tau] - K[\nu h, \tau]\} \quad \text{for } \nu h < u(t-\tau) < (\nu+1)h$$
$$= 0 \text{ elsewhere} \quad (53)$$

and

$$A_\nu(\tau) = (\nu+1)K[\nu h, \tau] - \nu K[(\nu+1)h, \tau] \quad \text{for } \nu h < u(t-\tau) < (\nu+1)h$$
$$= 0 \text{ elsewhere.} \quad (54)$$

On substituting (52) in (4) and inverting the order of summation and integration, the input-output relationship becomes

$$v(t) = \sum_{r} \left\{ \int_0^\infty W_r(\tau) u(t-\tau)\, d\tau + \int_0^\infty A_r(\tau)\, d\tau \right\} \tag{55}$$

where the $W_r(\tau)$ and $A_r(\tau)$ functions are given by (53) and (54). It is readily seen that this relation is a special case of (45) in which the

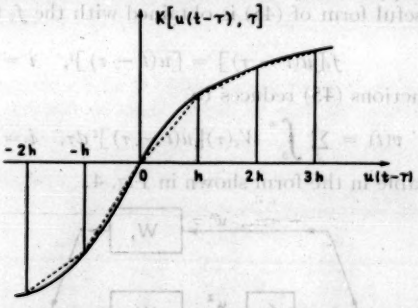

FIG. 5. Piece-wise linear approximation to the characteristic function.

f_i functions are expressed by

$$\begin{aligned} f_i[u(t-\tau)] &= u(t-\tau) + a_i \quad \text{for} \quad ih < u(t-\tau) < (i+1)h \\ &= 0 \text{ elsewhere,} \end{aligned} \tag{56}$$

where the constant a_i is given by

$$a_i = \frac{\int_0^\infty A_i(\tau)\, d\tau}{\int_0^\infty W_i(\tau)\, d\tau} \tag{57}$$

or more explicitly

$$a_i = \frac{\int_0^\infty \{\mathrm{K}[(i+1)h, \tau] - \mathrm{K}[ih, \tau]\}\, d\tau}{h \int_0^\infty \{(i+1)\mathrm{K}[ih, \tau] - i\mathrm{K}[(i+1)h, \tau]\}\, d\tau}. \tag{58}$$

The zero-memory nonlinear two-poles characterized by (56) can be mechanized in the form of suitably biased linear rectifiers. The canonical realization, then, assumes the form shown in Fig. 2, in which the boxes labeled $f_1, f_2, \cdots$ consist of biased linear rectifiers. The input-output relationship for the ith biased rectifier is expressed by (56), while the impulsive response of the ith linear two-pole is given by (53).

It is not difficult to devise many other canonical realizations for two-poles of class $\mathfrak{N}_1$. In general, these realizations are more complex than those discussed above.

RESPONSE WITH INITIAL EXCITATION

The problem considered in this section is that of finding the response of an initially excited two-pole of class $\mathfrak{N}_1$ to a specified input. For this purpose, it is convenient to work with the canonical realization shown in Fig. 2 in which it is assumed that the linear two-poles are of the lumped-constant type. This is equivalent to assuming that their system functions, $H_i(s)$, which are the Laplace transforms of the respective impulsive responses, $W_i(t)$, are of the form

$$H_i(s) = \frac{P_i(s)}{Q_i(s)} \tag{59}$$

where the $P_i(s)$ and $Q_i(s)$ are polynomials in s.

Suppose that a given input $u(t)$ is applied to a two-pole $N_{(1)}$ at $t = 0$, and that the initial state of excitation is specified in terms of the values of the output and its derivatives at $t = 0-$.

Since the nonlinear two-poles in the canonical realization of $N_{(1)}$ have no memory, the response of $N_{(1)}$ to $u(t)$ may be written as

$$v(t) = (\text{response of } N_{(1)} \text{ to } u(t) \text{ at rest}) + (\text{output of } N_{(1)} \text{ with } u(t) = 0). \tag{60}$$

The first term in this expression is simply

$$v_1(t) = \int_0^\infty \mathrm{K}[u(t-\tau), \tau]\, d\tau, \tag{61}$$

where $\mathrm{K}[u(t-\tau), \tau]$ is the characteristic function of $N_{(1)}$. As for the second term, we note that the output of $N_{(1)}$ in the absence of $u(t)$ is the same as the output of a linear two-pole which consists of the parallel combination of the linear two-poles in the realization of $N_{(1)}$ (Fig. 2). The impulsive response of this two-pole is the sum of the impulsive responses of the component linear two-poles,

$$W(t) = \sum_i W_i(t), \tag{62}$$

and its system function, $H(s)$, is given by

$$H(s) = \sum_i H_i(s). \tag{63}$$

Thus, the second term in (60) may be expressed as

$$v_2(t) = \text{output (with no input) due to initial excitation of the linear two-pole characterized by the system function } H(s). \tag{64}$$

It can be shown (7) that the output, $v(t)$, of a linear two-pole due to initial excitation alone, is given by

$$v(t) = \mathfrak{L}^{-1}\left\{\frac{1}{Q(s)}\left(\frac{Q(s) - Q(p_0)}{s - p_0}\right)v\right\} \tag{65}$$

where $\mathfrak{L}^{-1}$ denotes the inverse Laplace transformation, $Q(s)$ is the denominator of $H(s)$, and p_0 is the initial value operator defined by the operational relation

$$p_0^n v = v^{(n)}(0-) = \text{initial value of the } n\text{th derivative of } v(t) \text{ at } t = 0-. \tag{66}$$

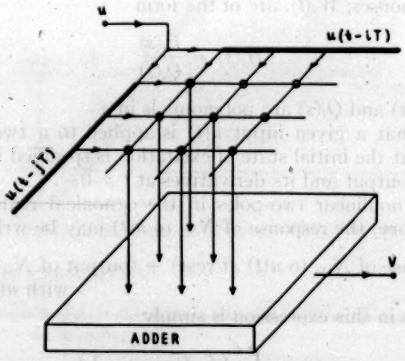

FIG. 6. Realization of a two-pole of class $\mathfrak{N}_1$ in the form of a two-dimensional delay-line filter. Superfluous nodes (because of symmetry) are indicated as circles. Note that the $(n \pm 1)$-poles lying on the main diagonal may be omitted.

On combining this result with (55) we obtain the following expression for the response of an initially excited two-pole of class $\mathfrak{N}_1$ to an input $u(t)$ applied at $t = 0$:

$$v(t) = \int_0^\infty \mathbf{K}[u(t-\tau), \tau]\,d\tau + \mathfrak{L}^{-1}\left\{\frac{1}{Q(s)}\left(\frac{Q(s) - Q(p_0)}{s - p_0}\right)v\right\}, \tag{67}$$

where to recapitulate

$u(t)$ = input to the two-pole, $N_{(1)}$, applied at $t = 0$
$\mathbf{K}[u(t-\tau), \tau]$ = characteristic function of the two-pole
$Q(s)$ = denominator of $H(s)$
$H(s) = \sum_i H_i(s)$
$H_i(s)$ = system function of the ith linear two-pole in the canonical realization of $N_{(1)}$ (Fig. 2)
p_0 = initial value operator defined by (66).

TWO-POLES OF CLASS $\mathfrak{N}_n$

The methods of characterization and realization of two-poles of class $\mathfrak{N}_n$, $n > 1$, are more involved but not different in principle from those discussed in the preceding sections. In what follows, two canonical realizations for two-poles of classes $\mathfrak{N}_n$ and $\mathfrak{N}_{n,1}$ (which is a subclass of $\mathfrak{N}_n$) are briefly outlined, and the additivity of two-poles of class $\mathfrak{N}_{n/1}$ with respect to certain types of inputs is established.

The canonical forms in question are essentially generalizations of those shown in Figs. 3 and 4. Thus, for a two-pole of class $\mathfrak{N}_n$ the analog of the input-output relationship (48) reads

$$v(t) = \sum_i \sum_j \cdots \sum_m \mathrm{K}_{ij\ldots m}[u(t-(i-1)\mathrm{T}), u(t-(j-1)\mathrm{T}), \cdots, u(t-(m-1)\mathrm{T})] \quad i, j, \cdots, m = 1, 2, \cdots. \quad (68)$$

This input-output relationship can be realized in the form of an n-dimensional tapped delay-line filter, as shown in Fig. 6 for $n = 2$, in which

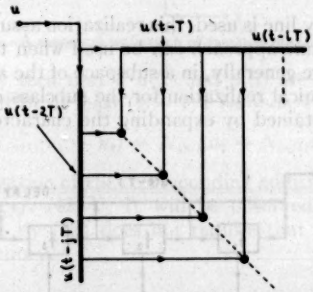

FIG. 7. Schematic representation of a two-pole of class $\mathfrak{N}_{2/1}$ as a degenerate form of two-pole of class $\mathfrak{N}_2$.

each axis consists of a tapped delay line, and the nodes represent zero-memory nonlinear $(n+1)$-poles having n input and one output terminals. These $(n+1)$-poles are such that the output of the $(n+1)$-pole to which the inputs are $u(t-(i-1)\mathrm{T}), u(t-(j-1)\mathrm{T}), \cdots, u(t-(m-1)\mathrm{T})$, is $\mathrm{K}_{ij\ldots m}[u(t-(i-1)\mathrm{T}), u(t-(j-1)\mathrm{T}), \cdots, u(t-(m-1)\mathrm{T})]$. The output of $N_{(n)}$ is the sum of the outputs of these $(n+1)$-poles. It will be noted that all those $(n+1)$-poles whose inputs are the same except for differences in order, can be replaced by a single $(n+1)$-pole.

From the physical point of view, there is no need to use n delay-lines since a single delay-line with appropriately interconnected taps can yield the same results. For purposes of visualization, however, it is perhaps more convenient to use the n-delay-line representation.

It is of interest to consider the realization of two-poles defined by the input-output relationship

$$v(t) = \int_0^\infty K[u(t-\tau), u(t-\tau-T), \cdots, u(t-\tau-(n-1)T), \tau]\,d\tau \quad (69)$$

where T is a constant. Such two-poles form a subclass of $\mathfrak{N}_n$ which will be denoted by $\mathfrak{N}_{n/1}$.

It is readily seen that a two-pole defined by (69) can be realized by utilizing only those nodes in the canonical realization of the general two-pole of class $\mathfrak{N}_n$ which lie on a straight line passing through the point $[u(t), u(t-T), \cdots, u(t-(n-1)T)]$ and making equal angles with the n axes. This is illustrated in Fig. 7 for a two-pole of class $\mathfrak{N}_2$, for which (69) reduces to

$$v(t) = \int_0^\infty K[u(t-\tau), u(t-\tau-T), \tau]\,d\tau. \quad (70)$$

If a single-delay line is used, this realization assumes the form shown in Fig. 8. The same approach can be used when the $(n+1)$-poles lie on a curve or, more generally, in a subspace of the n-dimensional space.

A second canonical realization for the subclass of two-poles defined by (69) can be obtained by expanding the characteristic function into

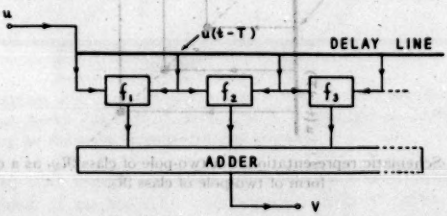

FIG. 8. Canonical realization of a two-pole of class $\mathfrak{N}_{2/1}$ in the form of a tapped delay-line with zero-memory nonlinear three-poles.

a power series in $u(t-\tau), u(t-\tau-T), \cdots, u(t-\tau-(n-1)T)$ and proceeding as in the case of two-poles of class $\mathfrak{N}_1$. This leads to the following input-output relationship:

$$v(t) = \sum_{i,j,\cdots,m} \int_0^\infty W_{ij\cdots m}(\tau)[u(t-\tau)]^i[u(t-\tau-T)]^j \cdots [u(t-\tau-(n-1)T)]^m\,d\tau \quad (71)$$

which is a generalized form of (51). The corresponding canonical realization is shown in Fig. 9, in which the box labeled cross-multiplier represents a device which generates terms of the form $[u(t)]^i[u(t-T)]^j \cdots$

$[u(t-(n-1)T)]^m$. These terms are applied to linear two-poles whose impulsive responses are the functions $W_{ij\cdots m}(t)$, and the outputs of these two-poles are summed in the adder, yielding the output, $v(t)$, expressed by (71).

One interesting property of two-poles of class $\mathfrak{N}_{n/1}$ is their additivity with respect to certain types of time-functions. Specifically, referring

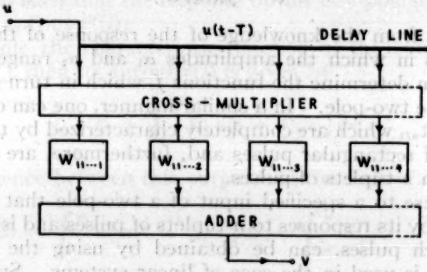

FIG. 9. A canonical realization of two-pole of class $\mathfrak{N}_{n/1}$.

to (69) it is readily seen that if $u_1(t)$ and $u_2(t)$ are two time-functions which are separated by at least $(n-1)T$ seconds, then

$$N_{(n/1)}(u_1+u_2) = N_{(n/1)}u_1 + N_{(n/1)}u_2, \tag{72}$$

which is a generalization of the corresponding additive property of two-poles of class $\mathfrak{N}_1$ [*cf.* (42)]. It will be observed that the additive property expressed by (72) does not require that the two-pole $N_{(n/1)}$ possess a finite memory.

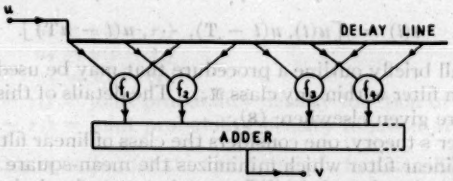

FIG. 10. A two-pole of class $\mathfrak{N}_{9/1}$ which is characterizable in terms of responses to pulse doublets.

It is of interest to inquire whether a two-pole of class $\mathfrak{N}_{n/1}$ can be characterized in terms of its responses to rectangular pulses or other forms of time-functions. While no definitive study of this problem has been made, it is not difficult to conceive of two-poles of class $\mathfrak{N}_{n/1}$ that are characterizable in this manner. For example, the two-pole shown in Fig. 10, which is of class $\mathfrak{N}_{9/1}$, is completely characterized by its

responses to doublets of rectangular pulses. Thus, if we consider a pair of rectangular pulses separated by T seconds, $\alpha_1\pi_h(t)$ and $\alpha_2\pi_h(t - T)$, of width h and amplitudes α_1 and α_2, then the response of the two-pole to such a pair is a train of rectangular pulses of width h and amplitudes v_i, such that for the ith zero-memory nonlinear three-pole,

$$v_i = f_i(\alpha_2, \alpha_1). \tag{73}$$

Consequently, from the knowledge of the response of the two-pole to pulse doublets in which the amplitudes α_1 and α_2 range over all real values, one can determine the functions f_i which in turn determine the structure of the two-pole. In a similar manner, one can construct two-poles of class $\mathfrak{N}_{n/1}$ which are completely characterized by their responses to n-tuplets of rectangular pulses and, furthermore, are additive with respect to such n-tuplets of pulses.

The response to a specified input of a two-pole that is completely characterized by its responses to n-tuplets of pulses and is additive with respect to such pulses, can be obtained by using the same general approach that is used in the case of linear systems. Specifically, the given input is first resolved into a set of pulse n-tuplets; then, the response of the two-pole to each n-tuplet is determined separately; and finally, these responses are added to yield the response of the two-pole to the specified input.

V. 19

OPTIMIZATION OF TWO-POLES OF CLASS $\mathfrak{N}_1$

The theory of optimum linear filters and predictors initiated by Wiener can readily be generalized to filters of class $\mathfrak{N}_n$. In his work on nonlinear transducers, Singleton (2) has treated the optimization of nonlinear filters whose input-output relationship is of the form

$$v(t) = f[u(t), u(t - T), \cdots, u(t - nT)]. \tag{74}$$

Here we shall briefly outline a procedure that may be used to determine the optimum filter within any class $\mathfrak{N}_n$. The details of this optimization procedure are given elsewhere (8).

In Wiener's theory, one considers the class of linear filters and determines that linear filter which minimizes the mean-square error; that is, the mean-square value of the difference between the desired output and the actual output of the filter. Such a filter is said to be optimum.

More generally, one may consider a class or a subclass of the system of classes $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$ and determine the optimum filter within such a class or subclass. For simplicity, we shall confine ourselves to the problem of finding the optimum filter within the class $\mathfrak{N}_1$.

Let N be such an optimum filter and let the input to N, $u(t)$, be a mixture of a stationary signal $s(t)$ and a stationary noise $n(t)$. The desired output, $v^*(t)$, is usually the signal $s(t)$ itself or $s(t + t_0)$, where t_0

is the prediction time. More generally, the desired output may be represented in the form

$$v(t) = \int_{-\infty}^{\infty} \mathrm{K}^*[s(t-\tau), \tau]\, d\tau, \tag{75}$$

where $\mathrm{K}^*[s(t-\tau), \tau]$ is the characteristic function of a fictitious two-pole of class $\mathfrak{N}_1$ such that the response of this two-pole to the signal $s(t)$ is the desired output $v^*(t)$.

If we denote the characteristic function of the optimum filter by $\mathrm{K}[u(t-\tau), \tau]$, then its output $v(t)$ is given by

$$v(t) = \int_{0}^{\infty} \mathrm{K}[u(t-\tau), \tau]\, d\tau, \tag{76}$$

and the difference between this output and the desired output, $v^*(t)$, is

$$v(t) - v^*(t) = \int_{0}^{\infty} \mathrm{K}[u(t-\tau), \tau]\, d\tau - \int_{-\infty}^{\infty} \mathrm{K}[s(t-\tau), \tau]\, d\tau. \tag{77}$$

It is the mean-square value of this difference that is minimized by the optimum filter.

On applying standard variational techniques to the minimization of (77), one finds that the characteristic function of the optimum filter satisfies the following linear integral equation:

$$\int_{0}^{\infty}\int_{-\infty}^{\infty} \mathrm{K}[u_2, \tau_2] p(u_1, u_2, \tau_1 - \tau_2)\, du_2\, d\tau_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{K}^*[s_2, \tau_2] p(u_1, s_2, \tau_1 - \tau_2)\, ds_2\, d\tau_2 \tag{78}$$

where

$u_1 = u(t - \tau_1)$
$u_2 = u(t - \tau_2)$
$s_2 = s(t - \tau_2)$
$p(u_1, u_2, \tau_1 - \tau_2)$ = joint probability density function of u_1 and u_2
$p(u_1, s_2, \tau_1 - \tau_2)$ = joint probability density function of u_1 and s_2.

To solve this equation for the characteristic function $\mathrm{K}[u(t-\tau), \tau]$ it is necessary, in general, to resort to machine computation. In essence, then, the foregoing analysis reduces the determination of the optimum filter within the class $\mathfrak{N}_1$ to a problem that can be handled by a computer.

It should be noted that, in order to determine the optimum filter within the class $\mathfrak{N}_1$, it is necessary to know the second order probability density functions for the signal and noise. More generally, it can be shown that the determination of the optimum filter within the class $\mathfrak{N}_n$ requires the knowledge of $2n$th order probability density functions for the signal and noise.

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NUCLEAR POWER AND INDUSTRY *

BY

JOHN J. GREBE [1]

SUMMARY

There are many compelling reasons for a privately financed reactor program, based on economic and military requirements. Dow-Edison do not expect to depend on the sale of the by-product, plutonium, for government use, although we do not overlook the need of such income. We expect that there will be a market for any excess fuel to meet the requirements for mobile reactors and of an expanding nuclear power industry. Our main objective is the development of a fast breeder reactor, providing high temperatures for power and using easily fabricated fuels, closely integrated with a low-cost separations process.

Our principal objective has been the selection of an approach consistent with long-range requirements but capable of early development. The reactor will be liquid-metal cooled and will use solid fuel elements. We believe that the best approach is early construction of a full-size unit. We assume that an exclusion area must be provided for the first reactor. Initially, only the reactor with integrated separations facilities would be built to minimize first costs. The heat produced could be readily discarded in such a location. Electrical generation and transmission facilities could be added later when a satisfactory reliability record for steam delivery has been established.

We have come to believe that the American way of life is the most rewarding way to man. History may well show, however, that the start of the decline in the American way of life began as a result of World War II. The reason is simple. In an effort to stay afloat in the present weapons race, we are using old materials and obsolescent methods on which we are squandering our defense dollars. At the same time we are destroying all private incentive for increasing American productivity by imposing upon ourselves an overwhelming tax load. The fruits of success are taxed away to support huge and wasteful Government-controlled operations.

To maintain the American way of life, industry must provide a large productive potential in the new fields of weapons production as it has in the past in all the other fields of manufacture, so that it can give us the capacity to make plowshares at the same time it gives us immense capacity for making swords. In the field of atomic energy, industry must put atomic reactor products into full production, suitable for both civilian and military uses, and thereby give to the people of the United States the resources of a peaceful, productive capacity, as well as the safeguard of military availability.

* Address delivered at the Stated Meeting of The Franklin Institute, January 21, 1953, in the Lecture Hall.

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According to the American way, this should be done by private industry, investing its own capital, using its highly developed skills and production methods, and earning a reasonable profit. These profits, if reasonably taxed, contribute to the support of Government. On the other hand, the continued expansion of Government-owned industries increases the tax load on private industries and all of the people, while the production of the Government-owned industries, instead of paying taxes, eats up the taxes of others.

The industrial potentialities of nuclear power are, therefore, forced on industry not primarily by the rosy picture of ultra light and low cost fuel anywhere on earth, or profits on bomb material made at a lower cost in private plants, but by something much more basic than all this. Private industry should be allowed to put the production of atomic reactor products on a profitable and tax-paying, rather than tax-consuming, basis. The alternative is national stagnation and decay. No realistic amount of spending for the production of either the old types of military equipment or the current atomic methods and materials can possibly keep up with the armament race. We need more than ten Hanfords—just as after World War I, we multiplied the vacuum tube and radio industry. We could never have had a big radio industry to put radar into large-scale production during World War II had the developments in radio communication during World War I been kept as a Government monoply.

You can readily see how the combination of inflation and taxes resulting from big spending destroys initiative at The Dow Chemical Company from the following: The Research Laboratory comes through with a beautiful process for doubling the production of a given product from a given plant. A new plant expansion would then require only one half the weight and size of equipment and facilities and half as much labor per pound of product as was needed twelve years ago for the same production. Certainly, that is a real achievement; it is better than the average which we have come to expect of industry, which, as a whole, average this much progress in 25 years.

The sales price of the product has gone up but very little in all these years; in fact, for the chemical industry, as a whole, it amounts to only about 20 per cent. The dollar cost of the plant, however, has gone up three-fold so that while in 1939, it cost only $12,000 per employee to build this plant, it now costs $36,000 per employee; but, of course, research has made it possible for the employee now to produce twice as much.

Now, let us find out how much we have to earn in order to make available the money for such construction.

Back in 1939, all one needed to earn was $15,000 in order to net, after taxes, the $12,000 for the capital required for one job. Now, to net after taxes the $36,000, we have to earn $200,000 more because of

the high tax bracket in which a successful company finds itself. The earning required before individual income taxes to make this capital available for one job is still higher if the company is compliant enough to pay part of the earnings as dividends instead of retaining all of the profits.

You can readily see now that it is exceedingly important for taxes to be at least low enough so that whatever rate of growth a healthy national economy requires in an active field can come about as a result of profits after taxes, otherwise the goose that laid the golden egg will die as soon as the hypodermic of necessity-certificates wears off. *It is not so much the loss of current resources that hurts the nation so severely as the abortion of new productivity that cannot pay off because of high taxes.* On an individual basis, it is not the sacrifice of all the recent raises in salary that hurts, but it is the loss of all incentive for new net increases that puts us on the skids.

Now that you know that our real purpose and object is to force consideration of more economical approaches toward obtaining the necessary superiority, based on typical American industrial productivity, how are we going to accomplish this? Back in 1948, Dr. Hafstad, AEC Director of Atomic Reactor Development, advised that the only way the military objectives that I outlined could be reached, would be for industrial atomic power to become a reality. Then, it seemed impossible that industry would dare to cut into a Government monopoly.

We have now arrived at a point where the Atomic Energy Commission has set up five industrial study groups actively pushing this program. The Dow-Edison group has the backing and cooperation of 28 companies, who have pooled experts in the field with sound business management and engineering judgment to get this job done as rapidly as possible.

How have we gone about it? When dealing with a very new field of science and engineering, one must be willing to discard all old concepts of what the new machinery should be like. By making an effort to look at the whole of the field of possibilities, one can become partly satisfied that the most simple and economical approaches for the conversion of nuclear energy to commercially useful forms have been considered.

An over-simplified outline of the combinations of fuels, their forms, some of the many variations of moderators, coolants, methods of energy conversion and power cycles to be considered is given in the following charts. These were charted out and evaluated on the basis of the following economic considerations:

1. Low cost reprocessing of fuel.
2. Frequent processing to minimize fission product inventory in reactor. (See Fig. 1.)

3. Minimum exclusion-area requirement or water-surrounded location.
4. High value utilization of fission-product heat.
5. Adaptability to prefabrication in shipyards and other shops.

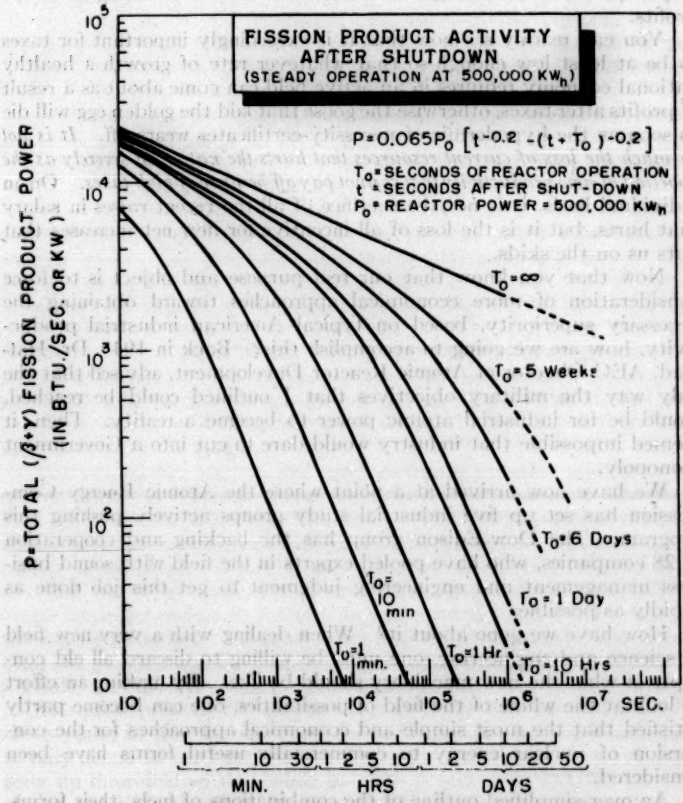
FISSION PRODUCT ACTIVITY
AFTER SHUTDOWN
(STEADY OPERATION AT 500,000 KW$_h$)
$P = 0.065 P_0 \left[t^{-0.2} - (t + T_0)^{-0.2} \right]$
T_0 = SECONDS OF REACTOR OPERATION
t = SECONDS AFTER SHUT-DOWN
P_0 = REACTOR POWER = 500,000 KW$_h$
P = TOTAL (β+γ) FISSION PRODUCT POWER (IN B.T.U./SEC. OR KW)
10^5
10^4
10^3
10^2
10
$T_0 = \infty$
T_0 = 5 Weeks
T_0 = 6 Days
T_0 = 1 Day
T_0 = 10 min
T_0 = 1 min.
T_0 = 1 Hr
T_0 = 10 Hrs
10 10^2 10^3 10^4 10^5 10^6 10^7 SEC.
1 10 30 1 2 5 10 1 2 5 10 20 50
MIN. HRS DAYS
t = TIME AFTER SHUT-DOWN (IN SECONDS)

FIG. 1.

6. Low cost fuel, that is, depleted uranium, natural uranium, or thorium.
7. High breeding gain, if possible, at optimum size.
8. High temperature operation to permit use of standard temperature steam-power cycle without sacrifice of reactor capacity.

9. Inherent self-regulation of reactor rather than dependence on external controls.
10. Speedy developments with risks of failure of individual approaches.
11. Several alternate expendable reactors to use the same fuel.
12. Thrifty use of standard materials of construction.
13. "Shoe string" but physically safe development program that is expected to be obsolete when it works.
14. Optimizing designs with realistic cost accounting, charging investments at a high enough rate to allow for growth out of profits after reasonable taxes on succeeding units.

The most important characteristic of a reactor has to do with the fuel that is consumed and the degree to which it is used up. A list of the fuels that have been discussed in the open literature is given in Table I arranged roughly in the order of the cost of the fuel isotopes

TABLE I.—*Nuclear Fuels Consumed.*

	Rating
Pu-239	F
U-235 (submarine or aircraft)	C
Nat. U + U-235	C
Nat. U partial burnup of U-235 content	C
Nat. U with burnup of U-235 + part U-238	C
Nat. U all consumed	G
U-238 all consumed	G
Th all consumed	G
Th all consumed + U-233 for sale	E
U-238 all consumed + Pu-239 for sale	E
Heavy metals consumed with accelerator	F
H_2 to He	F

with a very rough indication of its relative standing given in terms of *E* for economically sound; *G* for good; *C* for conditional; *F* for future possibility.

Table II gives an idea of some of the physical forms that the fuel may take. This is followed by some of the chemical forms that are most often mentioned. The combinations of all these forms that must be considered require large charts for their correlation.

Table III gives a list of the most generally mentioned elements and compounds used for moderators which are used to slow down the neutrons to bring them into the best range for fission reaction without excessive absorption in U-238 in the resonance energy range. This is the great invention of Dr. Fermi that made the original reactors possible. Included in this group should also be the change in energy in the upward direction such as when high energy neutrons or other high energy

particles are used to produce fission reaction by the use of particle accelerators of various types.

In Table IV are listed the agents capable of serving the purposes of cooling and transferring the heat. For our purposes, we have con-

TABLE II.—*Physical and Chemical Forms of Fuels.*

Physical Forms may be:
- Solid
- Sheet
- Rod
- Tubes
- Granules
- Clad or bare, suspended or packed.
- Liquid in various forms
  - As a solution, suspension, etc.

Chemical Compositions may be:
- Metal
- Metal alloy
- Metal oxides
- Fluorides
- Other metal salts
- Metal salts plus solvents

TABLE III.—*Moderators.*

Moderating Isotope	Usual Form	Rating
H_2	H_2O or other compound	G
D_2	D_2O or other compound such as OD^-	G
He	Compressed gas	C
Li^7	Oxide or hydride or other compound	C
Be	Metal or oxide	G
C	Dense graphite	E
O	Oxide	E
F	Fluoride	G
P	Element	C
Mg	Oxide or compound	C
Ca	Oxide or compound	C
Non-moderating diluents		
Pb		
Bi		
Zr		
Particle accelerators of various types		

centrated on liquids and slurries in order to avoid the high velocities that are associated with vapors when used to carry a given quantity of heat. Of these, metallic sodium and higher molecular weight materials such as tri- and tetraphenyl called Dowtherm C, water and mixtures of solid metals and liquids have been given the most consideration, with the first two being given the highest rating currently.

TABLE IV.—*Heat Transfer Agents.*

(All fuels and moderators should be considered as heat transfer agents)

Gas	Solid in Suspension	Liquid	Slurry	Solid	Mixtures of Solids
H_2	CH_4+C	H_2O	Na+Fe	Ice	Graphite
D_2	etc.	D_2O	etc.	Graphite	+Cu
He		Na		Carbon	etc.
Hydro-carbons		Hydro-carbons		Iron	
Freons		Dowtherms		Copper	
		Freon			
		Bi			
		Pb			

Table V gives the energy transfer methods that may be used for withdrawing nuclear energy from its source in a usable form. This listing should be given considerable discussion since one often hears about the awkwardness of converting 200 million-volt fission energy into ordinary thermal energy from which it is laboriously converted through turbines and generators into electrical energy. This process wastes about 70 per cent of the heat in current power plant operations.

(*a*) The most interesting and most efficient of all techniques for

TABLE V.—*Energy Transfer Methods.*

	Rating
(*a*) Particle emission	F
(*b*) Frequency conversion	F
(*c*) Conductance	E
(*d*) Mechanical oscillation	F
(*e*) Convection { natural	F
(*e*) Convection { inducted	E
(*f*) Change of state	E
(*g*) Heat of transformation and rearrangement	C
(*h*) Heat of dissociation	F
(*i*) Heat of reaction (reversible)	F
(*j*) Electromagnetic absorption	F
(*k*) Electrostatic absorption	F
(*l*) Thermoelectric currents	G

using nuclear energy would be the emission of particles in conjunction with the propulsion of inter-planetary vehicles or even the development of reaction-type rotating power generators depending upon the recoil of the particles when emitted. This, though theoretically sound, is something for my 7-year old boy to do when he grows up.

(*b*) Next, we deal with the development of a frequency converter that will change radiations with an enormously high frequency into the kind that we know how to use. This, too, is something for the future although this kind of an operation is understood and used as part of the techniques for measurements and control.

(*c*) Primary dependence upon conductance is a very practical method. The most simple form of such a unit is to generate the heat inside of a large graphite rod and to conduct it to the outside surface where it can be used for many purposes under good control.

(*d*) Next, while we are looking for unique ways of doing things, there is no reason why an ultra simple transfer of energy in the form of length or volume changes cannot be used. This oscillating source of energy can be turned on and off and be available in a wide range of temperature. Thus, atomic energy may well change our concepts of what it takes to produce power by using surges or alternate cycles of on and off. This, too, while opening up a new field was something I skipped over quite without any qualms of conscience six years ago and it is not regretted now.

(*e*) Convection induced by this new source of energy can have capabilities we have not found economical with ordinary chemical heat sources. Why not an atomic atmosphere to operate a windmill or to provide a waterfall? Can you imagine a more simple power system than the vaporization of lead at the bottom of a high column which then may lift more liquid lead and rise to the top where it can be condensed and run back to the bottom through an extremely compact and efficient turbine or Pelton wheel? So far we think that this process, while different, does not indicate good thermodynamic power-cycle efficiencies. It merely releases one from some problems of materials of construction. While this cycle works to produce circulation in a perculator or a steam boiler, we cannot even afford such forces working to help overcome the friction drop in many nuclear heat-transfer systems. Nature can afford it with solar heat, but we hope you will work it out for very low cost nuclear heat.

(*f*) In regard to the change of state method, heat transfer operations generally look for the added advantage of gaining heat capacity by the use of either the fusion of a solid into a liquid or the vaporization of a liquid.

(*g*) Heat of transformation and rearrangement due to changes from one crystal lattice to another and the like are small, but can have very important functions.

(*h*) A particularly favorable method capable of absorbing large quantities of heat is the use of heat of dissociation. For example, ethane can be broken up into ethylene and hydrogen. Many other illustrations may be given for this phenomena which should not be overlooked in energy-transfer problems.

(*i*) In addition to heat of dissociation, mentioned under (*h*), one should also consider reversible reactions involving heat of solution and heat of recombination. Molecular hydrogen can be changed to atomic hydrogen and back again, such as in the atomic hydrogen welding process. The best power cycle one could hope for would be one in which both heat of vaporization plus heat of dissociation would feed into the circuit so that for a given fluid with a given specific heat, one would transfer more energy and have it work through a wider range of temperature than one gets now from steam- and mercury-vapor power plants.

(*j*) Electromagnetic energy transfer may be a particularly convenient way to go directly from a fluid under pressure to an electrical generator without rotating parts; in fact, the combination of convection directly induced by nuclear heat in a perculator and a Faraday pump in reverse can convert the atomic energy directly into electrical power without rotating shafts. With the improvements possible when low temperature liquid conductors are used, it is hard to tell what to expect. An over-all efficiency of 15 per cent should be possible with what can be done in the near future.

(*k*) The most often referred-to short-cut in the conversion of atomic energy into useful work or electrical energy has to do with the possibilities of electrostatic energy absorption. This is particularly applicable to beta-emitting fission products. It is well known that the equivalent of a battery may be obtained by collecting the electrons discharged at a high voltage for producing a very low current electrical source. There is, however, an additional technique that may well be worth some limited application in which the electrostatic potential obtainable with beta-emitting sources is used to operate an electrostatic motor. This has the same relation to an electrometer that an ordinary electromagnetic motor has to a galvanometer.

(*l*) Thermoelectric currents have to do with the frequently mentioned idea of producing power direct from thermocouples. This is quite impractical for the kind of heat sources one normally deals with. However, in the case of nuclear reactors, we may have the advantage of a much higher temperature under very clean, non-oxidizing, non-slagging conditions. This may make graphite-thorium or graphite-zirconium thermocouples or many other lower cost combinations practical, with the resulting benefit of obtaining direct current without any kind of a commutator or rectifying device.

It may even be possible to intersperse this method of cooling be-

tween the original reactor heat source and a boiler tube used for the cold junction, so that additional power is obtained from a standard power plant, through the use of these thermoelectric currents. Nothing would please us more than to have someone develop this approach for commercial use.

In spite of some of the intriguing possibilities, we are aiming to produce power by the ordinary technique of circulating sodium in contact with the heat from the reactor. What a let-down! Again, we can say we are proposing to do the things that are practical, in a manner that we think we know how to carry out now, at a cost that will be competitive. Thus, the exact method by which the energy is transferred and transformed does not matter too much at this stage in the development.

TABLE VI.—*Motive Fluids and Power Cycles.*

(Consider the possibility of each also being a fuel, moderator, coolant and heat transfer agent.)

	Rating
H	C
H_2	C
He	C
H_2O	E
CO_2	C
Air	E
H_2F_2	C
Freons	F
SO_2	C
Hydrocarbons	F
Dowtherms	F
Hg	G
P_4	C
UF_6	C
UOF_4	C
Power Cycles:	
Hot gas cycles	
Rocket	
Ram jet	
Brayton Cycle	
Otto Cycle	
Rankine Cycle	
Single fluid	
Single fluid with dissociation	
Bifluid	
Superheat Cycle	

Finally, motive fluids and power cycles (see Table VI) are used to convert the heat that has been recovered at a sufficiently high temperature into electrical power. This field is again as big and open as it was in the original developments of power plants using the combustion of fuel for heat; in fact, it would be quite odd if 20 years from now, our big power plants would still be using water and steam. We can afford fluids in the new units that would have been too costly for the large inventory required in old-type steam boilers. Water, after all, is too good a solvent for use in power plants. In other words, if none of the short-cuts mentioned under the methods available for converting nuclear energy into usable form should come through, then it is most likely that our power plants will look very much like modern refrigerating units. Beyond listing the materials that have been considered particularly interesting for this purpose, we do not need to discuss them here. The maximum theoretical gain by the use of any of these might be to raise our thermal steam-cycle efficiency from 30 to approximately 40 per cent—hardly a big enough margin to work on when currently we are trying to substitute fuel at practically no cost for the big fuel bills of large power plants.

It is quite possible that these tables, made for such a survey six years ago, have overlooked the very best approaches. In fact, the only thing that gives us the courage to go on with our research and design work is the knowledge that when the current approaches are worked out in detail and commercially reproduced, their costs cannot help but be low enough to be competitive and that each of the components does not ask for any new industrial techniques. Obviously, it will take considerable time and effort to overcome the problems to be encountered.

If these thoughts teach us one thing, it is to look at all of our existing industry and national economy from the point of view of what it could be and will be when the effect of ample energy at a low cost anywhere on earth has taken hold.

Think of the ores that are shipped long distances to the source of coal or water power when they might just as well be processed close by, or wherever the climate, the fresh water supply and the moral tone and basic motives of the population permit new industries to grow. Think of the many cumbersome chemical, climatic and transportation problems that are faced all over the world for lack of a diversity of resources; yet air, water, clays, seawater and carbon in the form of coal, oil, gas or wood can be converted by low-cost power into almost all of the necessities—clothing, housing, transportation, recreation and education—that one might need. Even the remaining major physical requirement—food—can be produced in larger quantities at lower cost with the help of low-cost energy. Such problems as changing climates by pumping seawater into deserts, pumping fresh water to high, dry

areas and using the most economical power operated techniques for bringing our food to the table can be licked with current technology.

Needless to say, we have at least six favorites among all the millions of possible reactor combinations indicated in the tables. Three are in the field of central station application. We call them the pineapple, the cabbage head, and the radiator. We have chosen what has been called a very difficult and advanced approach. We think it is the only general direction in which one can make the products that the military should call for. In addition, it is a direction in which it may be possible to show commercially profitable results, independent of Government purchases.

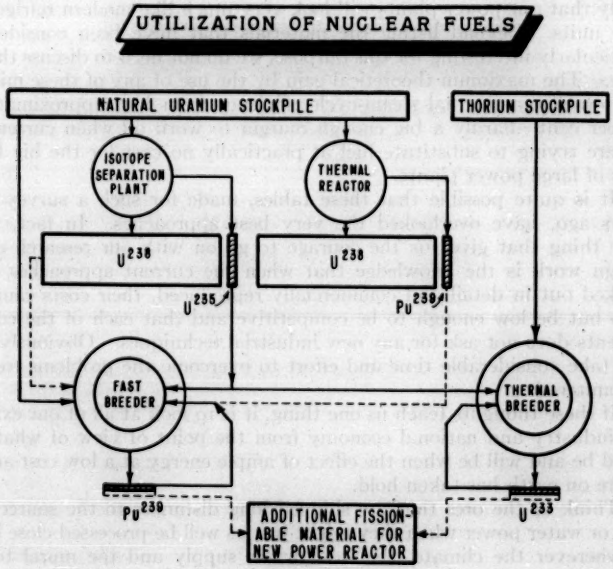

FIG. 2.

The nearest prototype of one of our proposed reactors is the Experimental Breeder Reactor.

Dr. W. H. Zinn, director of the AEC's Argonne National Laboratory, recently described the experimental "breeder" reactor at Arco, Idaho. It produces power by burning its own by-product, plutonium. The active core of the Arco breeder is about the size of a football. Our core will have to be much larger to generate several hundred times as much heat. It is hoped that Dr. Zinn's reactor will be able to

establish that breeding ordinary U^{238} into as much or more fissionable Pu^{239} as is consumed is possible, so as to verify the concept that all of natural U can be used for nuclear fuel. In the meantime, the uranium-fuel cost will be negligible. These 3.5 pounds will operate Detroit Edison for seven hours. With breeding, the world will have about 23 times as much energy available as is stored in all our coal, oil and gas. Thousands of years from now, when uranium and thorium will become scarce, we will know how to use hydrogen from water, to produce heat the way the sun does. (See Fig. 2.)

Dow-Edison expects, initially, that increased capital cost will eat up the whole of the fuel savings. The Detroit Edison Company has published their data showing that $225.00 of added capital per kilowatt-hour capacity without any other increases in cost anywhere would cost as much as the savings in fuel. This was calculated on the usual low rate of return on the capital in the utility industry, and on the assumption of no labor and maintenance cost. If we allow 30 per cent for depreciation, taxes and insurance, not too much for a new field in which obsolescence is very high, and if we achieve automatic operation with no labor for the "furnace room," the figure would be more nearly $130.00 per kilowatt for the new reactor and boiler equipment.

Still we struggle on, evaluating every new scheme proposed to reduce investment, improve yields and to increase the value of the products. From the beginning, we recognized that a by-product of power production, plutonium, could not be credited at what it costs the Government, or even what it may cost in the future. The most we are willing to count on as a credit for the bomb material now produced, is its value as an enriched fuel for mobile reactors and for stocking new stationary plants with the initial fuel charge. This requires a drastic reduction in costs.

As a guiding principle, we do not allow ourselves to think in terms of any nuclear reactor system which, although it may be relatively easy to build from an engineering standpoint, does not promise economical feasibility for the long run. Neither do we allow ourselves to think of nuclear power plants as economically competitive only with conventional plants having extra high fuel costs or with diesel-engine installations. Adherence to this policy makes the job more difficult but to forsake it would mean no solution to the problem of achieving the tremendous expansion for military needs.

To give you an idea of how we are economizing, we want to discuss those things not classified which bear on such economy and efficiency. For example, we know better than to go into an out-of-the-way place for an exclusion area. Power lines, roads, and time for transportation of people are very expensive, so if we can find a low-cost area close enough to power loads to be useful, we might well use it for the initial test installations. On the other hand, we cannot neglect such planning

and design that will prove that nuclear power plants can be more safe and less obnoxious than present plants. As an example, the exclusion area required by current regulations could be covered by highly productive truck gardens and greenhouses, warmed and watered by the warm cooling water from the atomic power plant. If exclusion areas are to be required, it is obvious that the land must somehow be brought to a higher usefulness to make it financially practical.

For a quick and reasonably easy start in the construction of facilities, we have another alternative, that of buying a surplus merchant ship and equipping it in a ship yard with a nuclear power reactor and facilities for disposing of the heat. In this case, we would not bother with a power line, to the ship or any turbo-electrical equipment, other than the facilities already installed on the ship.

By going off-shore from an industrial area near large high-voltage, existing power transmission lines, one could get the exclusion area that is required at least initially for the cost of the communication facilities to and from the ship. Later, when it has been established that present regulations calling for a large no-man's land are too severe and can be greatly relaxed due to improved methods of handling radioactive wastes, one will be able to locate nuclear plants much closer to high-power-consumption areas by putting them under ground or under water. Under these conditions, the primary hazard to be minimized or eliminated by engineering design would be that of enemy action causing a dispersion of radioactive materials from the reactor.

Making a full-scale test of a reactor would require only a little more of the expensive fuel than a test on a small scale, but it will then establish an experience record that may permit one to justify further expenses for turbo-generators, switch gear, and power lines when a satisfactory reliability record for steam generation has been established over a period of at least a year.

Economies do not stop as we approach something as old as steam-generation equipment. After all, the heat flow from the sodium which carries the heat out of the reactor to make superheated steam is temperature limited. It cannot overheat because, in itself, it is designed to go through a given temperature range each time around the cycle. This makes a big difference. It is like a double boiler that protects the food from scorching. Compare it to the heat-flow limitation that one has to contend with in present boilers. A plugged boiler tube in a coal- or oil-fired boiler burns out.

What is more, the ever-present ash and soot have forced a boiler design that consists of relatively large tubes requiring thick walls to withstand the abuses that such equipment has to take from slag, temperature shock and cleaning techniques. With a temperature-limited heat source having the thermal-conductivity of liquid metal on the outside of the tubes, one does not need to worry about oxidation,

slag and mechanical shock. Instead, it may be advantageous to make the tubes very small in diameter, possibly $\frac{1}{8}$ in., thus saving most of the wall thickness now required and reducing the weight and size of a boiler and superheater to considerably less than the size of the condenser currently used.

If, in addition, one takes advantage of an experimental boiler that we designed and built in Midland back in 1929 and operated as long as oil was an economical fuel until 1937, we can still further reduce cost, size, and complication by eliminating the vapor separating drum. This was a "once through" parallel tube type boiler and had a temperature-limited portion that was called upon to finish the boiling of the water and to superheat at 1500 psi. steam pressure with Dowtherm as the heating fluid. It worked well with $\frac{7}{8}$-in. inside diameter tubes. Now, thin bundyweld tubes, using multiple layer construction, offer the best guarantee for sound metal all over, for a job like this. Small headers would collect the steam into a number of delivery pipes leading to the turbine control valves. These are by-pass valves that divert the steam into de-superheaters and condensers rather than stop it. The boiler-feedwater valve admitting water to the flash boiler and superheater must be open at all times when the sodium is up to temperature to dispose of the heat whether there is a load on the turbine or not. In fact, for the most economical operation, a nuclear power plant should operate at a very high load factor because of the high fixed charges. It is just like running a hydro plant for all it is worth because there is always a surplus of water to spill.

The first time that any pressure vessel with any appreciable wall thickness should be required in the entire heat cycle is in the high pressure end of the turbine housing. All of the equipment from there on back to the source of heat can be made of thin walled materials in order to keep weight and size down to the minimum. Steam boilers of this type would actually be so small in size that they will be clustered near the turbine head and within a few feet of it. Eventually, when water and steam will be displaced by one of the synthetic fluids, such as are used in modern refrigerating units, we will still further reduce the size and costs of boilers, turbines and condensers.

This equipment would then be really compact. Only the necessary shielding against the radiation from the reactor coolant should separate the boilers from the turbine. Even this shielding against radiation, often consisting of 6 ft. of concrete and steel, cannot be constructed with expensive and permanent materials. It would be made up of multiple nested shells of sheet steel filled with wet sand or iron ore.

Ordinary materials of construction must be used to keep the costs down. Pumps must be the kind that have proven themselves efficient and economical by service in industry. We count on the usual lantern gland used throughout industry to permit a minimum of leakage of

fresh non-radioactive sodium into the circuit and out at the shaft in the semi-frozen form at the expense of a very small amount of loss each day.

We expect to employ new integrated methods of fuel processing to reduce costs enormously, although it would not constitute a serious set-back should such methods fail to operate well enough, for the simple reason that existing conventional processing plants stand by.

The plans for reactor construction require the use of low-cost materials that one can afford to discard or replace as often as necessary. We will expect considerable obsolescence initially because we do not intend to test each design or reactor element exhaustively under the illusion that we can predict and preclude all difficulties and failures. In fact, we propose to carry forward not less than three alternative reactor designs in order to keep ourselves from closing our minds to other possible techniques and to emphasize that we must not wait until we know all the answers before we go ahead. The nation cannot afford such delays.

If all these things do function and we manage to get rid of the bugs in a reasonably short time—one or two years—we believe that such a plant would become profitable and practical and worthy of multiplication. Then, we would have not only a badly needed military potential, paying taxes, but would also earn enough money to increase the number of such plants rather than taking from the earnings of other industries to boost this one. What a relief it will be to see atomic installations turning out power and taxes instead of using them up at most prodigious rates.

All this adds up to saying that we are confidently working on an approach to nuclear power generation which can stand on its own feet without Government subsidy or military markets. We feel it is necessary to aim for the best that one can see on the horizon if we are to have enough of what it takes to forestall or win a major conflict. We expect that our work will help to make up for some of the gaps in the national defense program, and make it possible to drop obsolete methods of warfare that require us to match man against man on the battlefield.

All this is to illustrate the commercial approach of speed and low cost, as opposed to the usual requirements of Government operations in which the certainty of easily achievable results, regardless of cost is the prime requisite. Put yourself into the state of mind of the Egyptian ruler over an early steel maker 3000 years ago. He soon found out that he needed a labor-saving use for steel to make men available for increasing steel capacity. We, too, have reached the point in atomic developments where it is essential to apply the new knowledge on making steel for swords and armor to produce plenty of plowshares to replace digging with sticks and bones. Without adequate industrial capacity, we would not have enough capacity to produce the

newer weapons so as to prevail if the showdown should come. We would then have to fall back on the bow and arrow-like weapons of the last war and possibly suffer a severe set-back in the advance of our way of life.

All through a recent presentation of these thoughts before a group of young graduate students, I could feel them cringe inwardly. Some of you here may have reacted the same way. You wish that somehow the human mind could repeal some of the laws of nature. Can't atomic energy turn out to be the soothing oil on troubled waters, make up for our past economic sabotage and provide such plenty that there need be no further jealousy or strife? Why must we use so much mental and physical effort to oppose amoral cunning, military, psychological and political power, and overwhelming superiority in number of soldiers? Why must we employ our superior scientific and productive power to make up for the much smaller number of our men? Why can't men agree to be guided by the Golden Rule?

The sad fact is that the Golden Rule has not been and cannot be adequate to cope with these problems. Even the high standard of living and learning in Germany as compared to its victims, both in 1914 and 1939, didn't prevent war. You can't buy friendship. There have been many idealistic, socialistic and communistic communities fostered even with large endowments and governmental support. They always fail, even on a small scale where people know one another well. They fail for lack of self-discipline, or self-denial for the sake of future improvement. There always have to be men who save and build and plan for the future in order that we as a whole can be fruitful and multiply.

We know that to find our way out of this moral dilemma our primary motive must be the love of God and the guidance of His spirit. And we must be free of self-righteousness. Almost all of the evils of recent history—from the pious efforts of pink do-gooders giving other people's money, to the ruthless and brutal inhumanity of the totalitarian tyranny—can be ascribed to self-righteousness. But since we are faced with the murderous threat of aggression from a foe bound by no moral considerations and assailable in his self-righteousness by no appeal to reason, our only present material defense is our technological skill and our production potential.

With this material defense we must strive to couple a spiritual quality which will arise in men with an active and alert conscience, not in well-kept human cattle with cradle-to-grave security—a security which approximates that of a man in prison. Leadership in American industry will be capable of strengthening our material defense if it looks beyond temporary pleasure and comfort to a long-range cooperation with creation as a whole, in its effort to produce more noble beings—more souls who love God and his creation.

The alternative to respecting the dignity of the individual—the alternative to the individual's being a good steward of his capabilities—is to expose the world to dangers of immense human woe.

No finer statement of this situation can be found than that of Thomas E. Murray, one of the members of our U. S. Atomic Energy Commission. His philosophy was stated in these words in a recent address:

> We have never had anything to fear from God's material creation. Our only fear and our great responsibility is not what we do with things, but rather with ourselves. Our only hope to keep the atomic energy basket from becoming, like Pandora's box, a source of immense human woe, is for each and every one of us to find in every majestic scientific discovery additional reasons for loving God more, and our neighbor as ourselves.

THE ALGEBRAIC THEORY OF LINEAR TRANSMISSION NETWORKS *

BY

MAYNARD G. ARSOVE [1]

Part II [2]

7. ITERATIVE IMPEDANCES

The concept of "iterative impedance" is intrinsically related to those of "load impedance" and "input impedance," which we now define.

Definition 15. A transmission network $t' = \chi t + \tau$ will be said to be terminated in a *load impedance* Z_L (*load admittance* Y_L) provided the output emf and current vectors satisfy $e' = Z_L i'$ for arbitrary i' ($i' = Y_L e'$ for arbitrary e'), where Z_L (Y_L) is an $n \times n$ matrix.

If termination of a given transmission network N in a load impedance Z_L (load admittance Y_L) implies that the input emf and current vectors satisfy an affine transformation of the form $e = Z_I i + \varepsilon_I$ for arbitrary i, then the $n \times n$ matrix Z_I is called an *input impedance* of N corresponding to the load impedance Z_L (load admittance Y_L). When no mention is made of the load impedance or admittance, it will be assumed that we are dealing with the case of a load admittance $Y_L = 0$.[10]

Uniqueness of the input impedance (when it exists) for a given load impedance (or admittance) is immediately evident.

Definition 16. If a passive transmission network N has input impedance λ when terminated in a load impedance λ, then λ is called an *iterative impedance* of N.

In general, iterative impedances are not unique, since the unit element, for example, has any load impedance as an iterative impedance.

Theorem 8. If a passive transmission network N in Γ has matrix

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix},$$

then the iterative impedances of N are characterized as the solutions λ of the $n \times n$ matrix equation

$$\lambda\gamma\lambda + \lambda\delta - \alpha\lambda - \beta = 0. \tag{15}$$

Proof. It is easily seen that every iterative impedance λ must satisfy (15), since $e' = \lambda i'$ and $e = \lambda i$ result in

$$(\lambda\gamma\lambda + \lambda\delta - \alpha\lambda - \beta)i = 0$$

for arbitrary i.

On the other hand, suppose that λ is a solution of (15). It then follows that $\alpha - \lambda\gamma$ is non-singular, since if it were singular we could find a non-zero row

* Research conducted while the author was recipient of a National Research Council Predoctoral Fellowship.

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[2] Part I appeared in this JOURNAL for April, 1953, p. 301.

[10] This convention corresponds physically to leaving the output terminals unconnected.

NOTE: Footnotes 3–9 are found in the text of Part I of this paper.

vector x satisfying $x(\alpha - \lambda\gamma) = 0$ and therefore $x(\beta - \lambda\delta) = 0$; but this leads to $x(U_n, -\lambda)\chi = 0$, from which the non-singularity of χ forces $x = 0$. Now, terminating N in a load impedance λ yields $e' = \lambda i'$, and from (15) we find

$$(\alpha - \lambda\gamma)(e - \lambda i) = 0.$$

Hence, by the non-singularity of $\alpha - \lambda\gamma$ there follows $e = \lambda i$, so that λ must be an iterative impedance of N.

Let us examine the solution of (15) for a few special cases. *If N is a series impedance transmission network other than the identity, then N does not admit an iterative impedance.* This is obvious from the expression for χ as

$$\chi = \begin{pmatrix} U_n & \beta \\ 0 & U_n \end{pmatrix};$$

since Eq. 15 then reduces to $\beta = 0$. Also, *if N is a parallel impedance transmission network other than the identity, then N does not admit a non-singular iterative impedance.* The transmission matrix in this case is

$$\chi = \begin{pmatrix} U_n & 0 \\ \gamma & U_n \end{pmatrix},$$

so that (15) becomes $\lambda\gamma\lambda = 0$. Finally, we note that *if N is a passive impedance-symmetric transmission network having 0 as an iterative impedance, then N is the series combination of a parallel impedance transmission network and a transformer bank.* In fact, $\lambda = 0$ forces $\beta = 0$, and the impedance-symmetry of N results in $\delta = (\alpha^*)^{-1}$.

Although the general solution of Eq. 15 is rather complicated, there is available for symmetrical sections a refinement of this equation which leads to a simple characterization of λ whenever γ is non-singular.

Theorem 9. Let N be a passive symmetrical section with transmission matrix

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}.$$

Then a necessary condition for λ to be an iterative impedance of N is that it satisfy

$$\left.\begin{aligned} (\lambda\gamma)^2 &= \alpha^2 - U_n \\ \text{and} \quad (\lambda\gamma)\alpha &= \alpha(\lambda\gamma) \end{aligned}\right\}. \tag{16}$$

Moreover, if γ is non-singular, this condition is also sufficient.

Proof. Post-multiplication of (15) by γ and application of (14) lead to

$$(\lambda\gamma)^2 + (\lambda\gamma)\alpha - \alpha(\lambda\gamma) - (\alpha^2 - U_n) = 0, \tag{g}$$

which for γ non-singular is equivalent to (15). Thus,

$$(\lambda\gamma - \alpha)(\lambda\gamma + \alpha) = -U_n,$$

and we have

$$\alpha = (1/2)[(\lambda\gamma + \alpha) + (\lambda\gamma + \alpha)^{-1}].$$

It is clear from this that α must commute with $\lambda\gamma + \alpha$ and hence with $\lambda\gamma$; that is, $(\lambda\gamma)\alpha = \alpha(\lambda\gamma)$. Equation (g) then reduces to $(\lambda\gamma)^2 = \alpha^2 - U_n$. Conversely, the last two equations imply (g).

Condition (16) is simply that the matrix $\alpha^2 - U_n$ admit a square root commutative with α. In this connection we note (6, p. 31)[11] that a square matrix need not have a square root, as for example the 2×2 matrix

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

Furthermore, even when $\alpha^2 - U_n$ does admit a square root x, x may fail to commute with α; for example

$$\alpha = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Corollary 9.1. Let N be a passive symmetrical section for which γ is non-singular. Then a necessary and sufficient condition that N admit an iterative impedance is that $\alpha^2 - U_n$ admit a square root commutative with α. Moreover, every iterative impedance λ of N is of the form

$$\lambda = \sqrt{\alpha^2 - U_n}\gamma^{-1},$$

where $\sqrt{\alpha^2 - U_n}$ is such a square root.

The question as to whether every iterative impedance of an impedance-symmetric transmission network is symmetrical is answered in the negative by citing the case of

$$\alpha = \gamma = U_2 \quad \text{and} \quad \lambda = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

However, we can make the following assertion.

Theorem 10. Let N be a passive symmetrical section for which γ is non-singular, and let λ be an iterative impedance of N. If N is impedance-symmetric, then λ^* is also an iterative impedance of N.

Proof. We have $\lambda = \sqrt{\alpha^2 - U_n}\gamma^{-1}$, where α and $\sqrt{\alpha^2 - U_n}$ commute. Then

$$\lambda^* = (\gamma^{-1})^*(\sqrt{\alpha^2 - U_n})^* = [\gamma^{-1}(\sqrt{\alpha^2 - U_n})^*\gamma]\gamma^{-1},$$

since by theorems 3 and 5

$$(\gamma^{-1})^* = \alpha\gamma^{-1}\delta - \beta = (\alpha\gamma^{-1}\delta\gamma - \beta\gamma)\gamma^{-1} = (\alpha^2 - \beta\gamma)\gamma^{-1} = \gamma^{-1}.$$

It remains to show that $\gamma^{-1}(\sqrt{\alpha^2 - U_n})^*\gamma$ is a square root of $\alpha^2 - U_n$ commutative with α. But, from

$$\gamma^{-1}\alpha^*\gamma = (\gamma^{-1})^*\alpha^*\gamma = \alpha\gamma^{-1}\gamma = \alpha$$

we obtain

$$[\gamma^{-1}(\sqrt{\alpha^2 - U_n})^*\gamma]^2 = \gamma^{-1}[(\alpha^*)^2 - U_n]\gamma = \alpha^2 - U_n.$$

Further,

$$\alpha[\gamma^{-1}(\sqrt{\alpha^2 - U_n})^*\gamma] = \gamma^{-1}\alpha^*(\sqrt{\alpha^2 - U_n})^*\gamma = \gamma^{-1}(\sqrt{\alpha^2 - U_n})^*\alpha^*\gamma$$
$$= [\gamma^{-1}(\sqrt{\alpha^2 - U_n})^*\gamma]\alpha,$$

and the proof is complete.

[11] The boldface numbers in parentheses refer to the references appended to Part II of this paper.

8. SOME REMARKS ON FUNCTIONS OF MATRICES

For the practical calculation of matrix functions, as well as for use in the development of our theory, we have recourse to the following classical theorem[12] on the reduction of a matrix to canonical form: *if X is an $n \times n$ matrix, there is a non-singular $n \times n$ matrix T such that*

$$T^{-1}XT = \begin{bmatrix} R_1 & & & \\ & R_2 & & \\ & & \ddots & \\ & & & R_k \end{bmatrix}, \tag{17}$$

where each R_j is of the form $R_j = r_j + s_j$ with r_j a scalar matrix whose diagonal elements are a characteristic root of X and s_j a matrix of the form

$$s_j = \begin{bmatrix} 0 & \epsilon & & & \\ & 0 & \epsilon & & \\ & & \ddots & \ddots & \\ & & & \ddots & \epsilon \\ & & & & 0 \end{bmatrix} \tag{18}$$

in which either $\epsilon = 0$ or $\epsilon = 1$. A matrix of the type $T^{-1}XT$ is called a *canonical form* of X, and each of the submatrices R_j is termed a *canonical submatrix* corresponding to X. From the theory of reduction to canonical form we know that the canonical submatrices are uniquely determined by X provided we make the further restriction that no characteristic root appears as the diagonal elements of more than one scalar canonical submatrix.

Our first application of canonical forms is to the determination of a square root of $\alpha^2 - U_n$ commutative with α, which plays a central role in the construction of iterative impedances.

Theorem 11. If α is an $n \times n$ matrix for which no non-scalar canonical submatrix has diagonal elements ± 1, then $\alpha^2 - U_n$ admits a square root commutative with α.

Proof. Let $\alpha_c = T^{-1}\alpha T$ be a canonical form of α, so that

$$\alpha_c^2 - U_n = T^{-1}(\alpha^2 - U_n)T.$$

Further, let us discard the subscript j and take $R = r + s$ as any canonical submatrix of α_c. Then corresponding to R in $\alpha_c^2 - U_n$ there is the submatrix

$$R^2 - U = (r^2 - 1) + (2r + s)s.$$

If $R^2 - U = 0$, we take its square root as 0. Otherwise, we expand $[(r^2 - 1) + (2r + s)s]^{\frac{1}{2}}$ by the binomial theorem, noting that $r^2 - 1 \neq 0$ and that *the expansion terminates* in view of the nilpotency of s. In each case the resulting square root $\sqrt{R^2 - U}$ commutes with R.

[12] See 4, pp. 34–37; 5, chapter 4; and 6, chapter 3.

Replacing every submatrix $R^2 - U$ by the corresponding matrix $\sqrt{R^2 - U}$ yields a square root of $\alpha_c^2 - U_n$ which we denote by $\sqrt{\alpha_c^2 - U_n}$. Finally, we set

$$\alpha^2 - U_n = T\sqrt{\alpha_c^2 - U_n}T^{-1}$$

and observe that this is actually a square root of $\alpha^2 - U_n$ commutative with α.

We next summarize a few results in the theory of matrix power series with scalar coefficients.[13] Let $p(x) = \sum_{k=0}^{\infty} a_k x^k$ denote a power series with radius of convergence ρ. Then *the matrix power series formed with the same coefficients converges when all characteristic roots have modulus less than* ρ. Moreover, for $R = r + s$ a typical $m \times m$ canonical submatrix with $|r| < \rho$ we have

$$p(R) = p(r) + p'(r)s + \cdots + [1/(m-1)!]p^{(m-1)}(r)s^{m-1}.$$

Setting aside the trivial case of $s = 0$, we can visualize this matrix in terms of its elements as

$$p(R) = \begin{bmatrix} p(r) & p'(r) & \frac{1}{2!}p''(r) & \cdots & \frac{1}{(m-1)!}p^{(m-1)}(r) \\ & p(r) & p'(r) & \cdots & \\ & & p(r) & \cdot & \\ & & & \cdot & \\ & & & \cdot & \\ & & & \cdot & p'(r) \\ & & & & p(r) \end{bmatrix}.$$

If $Y = T^{-1}XT$ is a canonical form of X, then $p(Y)$ is obtained by replacing each canonical submatrix R by $p(R)$, and $p(X)$ is given by

$$p(X) = Tp(Y)T^{-1}.$$

It is evident from these remarks that the exponential, trigonometric, and hyperbolic functions can be defined for all square matrices, and that inverses of these functions exist provided the characteristic roots are suitably restricted.

9. THE CHARACTERISTIC IMPEDANCE

In studying transmission networks it is of interest to consider the series combination of m identical passive sections and examine the behavior of the input impedance as $m \to \infty$.

[13] Further details are to be found in 5, chapter 6 (Sections 6 and 7) and 6, chapter 8.

Definition 17. Let N be a passive transmission network for which N^m admits an input impedance $Z(m)$ for all sufficiently large integers m. If $\lim_{m\to\infty} Z(m)$ exists, it will be called the *characteristic impedance* of N.

Whenever a characteristic impedance exists, it will be seen to constitute a fundamental parameter associated with the given transmission network.

From the definition of input impedance it is evident that a passive transmission network with transmission matrix

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

admits an input impedance if and only if γ is non-singular, the input impedance being determined as $Z = -\gamma^{-1}\delta$.

Theorem 12. If N is an impedance-symmetric transmission network admitting a characteristic impedance λ_0, then $\lambda_0^* = \lambda_0$.

Proof. Since the passive impedance-symmetric transmission networks form a group, the transmission network $N' = N^m$ is passive and impedance-symmetric. By (*b*) of theorem 3 we see that its input impedance $Z(m) = -(\gamma')^{-1}\delta'$ is symmetrical, and the theorem follows by letting $m \to \infty$.

A classical property of the characteristic impedance is the following.

Theorem 13. If a transmission network N admits a characteristic impedance λ_0, then λ_0 is also an iterative impedance of N.

Proof. It is obvious that if N is combined in series with N^m, then N has output impedance $Z(m)$ and input impedance $Z(m+1)$. In the limit as $m \to \infty$ we therefore have $e' = \lambda_0 i'$ and $e = \lambda_0 i$.

However, the existence of an iterative impedance does not imply the existence of a characteristic impedance. For example, the symmetrical T-section of Fig. 10, where j denotes the imaginary unit and ω, L, C

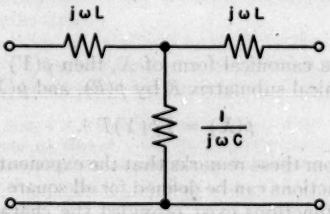

FIG. 10.

satisfy $\omega L = 1/(\omega C)$, has $\lambda = \pm \omega L$ as iterative impedances. We find also that

$$Z(m) = 0 \quad (m \text{ odd})$$
$$= \infty \quad (m \text{ even}),$$

which renders impossible the existence of a characteristic impedance.

We now proceed to develop a criterion for the existence of a characteristic impedance, as well as a method for constructing the characteristic impedance when it exists, for the case of symmetrical sections.

Lemma 2. If a symmetrical section with transmission matrix

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

admits a characteristic impedance, then γ is non-singular.

Proof. We show that the matrix γ' in

$$\chi^m = \begin{pmatrix} \alpha' & \beta' \\ \gamma' & \delta' \end{pmatrix}$$

always has γ as a post-factor. This is trivial for $m = 1$. The assumption that it is valid for a particular m, that is that $\gamma' = c\gamma$, yields its validity for $m + 1$, since

$$\chi^{m+1} = \begin{pmatrix} * & * \\ \gamma'\alpha + \delta'\gamma & * \end{pmatrix}$$

and $\gamma'\alpha + \delta'\gamma = c\gamma\alpha + \delta'\gamma = (c\delta + \delta')\gamma$ by (14). Thus it follows by induction that γ' always has γ as a post-factor. Existence of a characteristic impedance implies the non-singularity of γ' for m sufficiently large, and this in turn implies the non-singularity of γ.

From corollary 9.1 and theorem 13 it is apparent that the characteristic impedance has the form $\sqrt{\alpha^2 - U_n}\gamma^{-1}$, where the indicated square root commutes with α. However, the problem of determining the *particular* square root which singles out the characteristic impedance from among the other iterative impedances is non-trivial.

We proceed to introduce a "principal square root" of $\alpha^2 - U_n$ and show that this leads to a solution of the problem. The key to the situation lies in the fact that a necessary and sufficient condition for the complex number $r^2 - 1$ to have a non-zero square root located either in the interior of the same quadrant or on the same quadrantal boundary as r in the complex plane is that $|r| > 1$ hold whenever r is real. Such a square root will be called a *principal square root of* $r^2 - 1$. This notion extends at once to canonical submatrices $R = r + s$. Whenever r is not real and in absolute value ≤ 1, we define the *principal square root of* $R^2 - U$ as that obtained by using the principal square root of $r^2 - 1$ in the binomial expansion of $[(r^2 - 1) + (2r + s)s]^{\frac{1}{2}}$ (see the proof of theorem 11).

The general definition is then framed in terms of this, by reducing the matrix to canonical form.

Definition 18. Let α be an $n \times n$ matrix reducible by a transformation T to the canonical form

$$T^{-1}\alpha T = \begin{pmatrix} \alpha_1 & 0 \\ 0 & E \end{pmatrix},$$

where α_1 is an $m \times m$ submatrix and E is a diagonal matrix of elements ± 1 (we allow the possibility of either α_1 or E being vacuous). Further, let all real charac-

teristic roots of α_1 be in absolute value >1, and let β be the $n \times n$ matrix obtained by replacing the submatrices of

$$\begin{pmatrix} \alpha_1^2 - U_m & 0 \\ 0 & 0 \end{pmatrix}$$

by their principal square roots, as defined above. Then the matrix $T\beta T^{-1}$ will be called the *principal square root* of $\alpha^2 - U_n$.

When it exists, the principal square root of $\alpha^2 - U_n$ is clearly unique and is a square root of $\alpha^2 - U_n$ commutative with α.

With this in hand we can formulate the existence theorem for characteristic impedances as follows.

Theorem 14. Let N be a passive symmetrical section with transmission matrix

$$\chi = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}.$$

Then for N to admit a characteristic impedance it is necessary and sufficient that γ be non-singular and that $\alpha^2 - U_n$ admit a principal square root. Moreover, the characteristic impedance λ_0 is (uniquely) determined as

$$\lambda_0 = -\sqrt{\alpha^2 - U_n}\,\gamma^{-1},$$

where the indicated square root is the principal square root of $\alpha^2 - U_n$.

Proof. (*Sufficiency*). Since γ is non-singular, we have

$$\chi = \begin{pmatrix} \alpha & (\alpha^2 - U_n)\gamma^{-1} \\ \gamma & \gamma\alpha\gamma^{-1} \end{pmatrix}.$$

Then, for $\alpha_c = T^{-1}\alpha T$ a canonical form of α there results

$$\chi = \begin{pmatrix} T & 0 \\ 0 & \gamma T \end{pmatrix} \begin{pmatrix} \alpha_c & \alpha_c^2 - U_n \\ U_n & \alpha_c \end{pmatrix} \begin{pmatrix} T & 0 \\ 0 & \gamma T \end{pmatrix}^{-1}. \tag{19}$$

Our hypotheses also allow us to express α_c as

$$\alpha_c = \begin{pmatrix} \alpha_1 & 0 \\ 0 & E \end{pmatrix},$$

according to definition 18.

Defining ϑ as that determination of $\cosh^{-1}\alpha_1$ having the same block form as α_1 and having characteristic roots with positive real parts, we have

$$\cosh\vartheta = \alpha_1$$

and

$$\sinh^2\vartheta = \alpha_1^2 - U.$$

The motivation for our definition of the principal square root now becomes apparent, for it is precisely this square root which yields

$$\sinh\vartheta = \sqrt{\alpha_1^2 - U}.$$

In fact, the following considerations suffice to determine $\sinh\vartheta$ from among the square roots of $\alpha_1^2 - U$ as the *principal* square root: (1) $\sinh\vartheta$ is triangular (zeros below the main diagonal) and has the same block form as α_1; (2) the characteristic roots of $\sinh\vartheta$ coincide, respectively, with those of the principal square root. Assertion (2) follows from the fact that the characteristic roots of $\sinh\vartheta$ lie in the same quadrant (or on the same quadrantal boundary) as the corresponding characteristic

roots of α_1; and this, in turn, is evident from the observation that for $r = x + iy$ a characteristic root of ϑ we have $x > 0$ and

$$\cosh r = \cosh x \cos y + i \sinh x \sin y$$
$$\sinh r = \sinh x \cos y + i \cosh x \sin y.$$

There results

$$\chi = \begin{pmatrix} T & 0 \\ 0 & \gamma T \end{pmatrix} \begin{pmatrix} \cosh \vartheta & 0 & \sinh^2 \vartheta & 0 \\ 0 & E & 0 & 0 \\ U & 0 & \cosh \vartheta & 0 \\ 0 & U & 0 & E \end{pmatrix} \begin{pmatrix} T & 0 \\ 0 & \gamma T \end{pmatrix}^{-1},$$

and, by induction, we find that χ^m is given by a similar expression but with the second matrix of the right-hand member replaced by

$$\begin{pmatrix} \cosh m\vartheta & 0 & (\sinh m\vartheta) \sinh \vartheta & 0 \\ 0 & E^m & 0 & 0 \\ (\sinh m\vartheta)(\sinh \vartheta)^{-1} & 0 & \cosh m\vartheta & 0 \\ 0 & mE^{m-1} & 0 & E^m \end{pmatrix}.$$

The non-singularity of $\sinh \vartheta$, which enters here, is of course a consequence of the fact that α_1 does not have ± 1 as a characteristic root.

A simple calculation reveals the input impedance of N^m to be

$$Z(m) = -T \begin{pmatrix} (\sinh \vartheta) \coth m\vartheta & 0 \\ 0 & (1/m)E \end{pmatrix} T^{-1}\gamma^{-1}. \tag{20}$$

Since all characteristic roots of ϑ have positive real parts, $\lim_{m \to \infty} \coth m\vartheta$ exists as the identity matrix, and there follows

$$\lambda_0 = -T \begin{pmatrix} \sinh \vartheta & 0 \\ 0 & 0 \end{pmatrix} T^{-1}\gamma^{-1} = -T\sqrt{\alpha_e^2 - U_n} T^{-1}\gamma^{-1}$$
$$= -\sqrt{\alpha^2 - U_n}\gamma^{-1},$$

the square root being the principal square root.

(*Necessity*). That γ must be non-singular has been established in lemma 2, so we can start with the representation (19). However, all that can be said of the matrix α_1 in this case is that it has no diagonal submatrix of elements ± 1 (these elements comprise the matrix E). We proceed to show that $\alpha^2 - U_n$ admits a principal square root, by proving that no real characteristic root r of α_1 satisfies $|r| \leq 1$.

Let us examine first the possibility of having a real characteristic root r with $|r| < 1$ and no characteristic root equal to ± 1. In this case we can apply the preceding development as far as (20). The divergence at this point is due to the fact that the characteristic root of ϑ corresponding to r has real part 0, so that coth $m\vartheta$ does not approach a limit. In fact, the submatrix of coth $m\vartheta$ corresponding to r has oscillatory characteristic roots, and an elementary argument shows that this oscillation must appear also in $Z(m)$, preventing the existence of a limit. Hence, this case cannot occur.

It remains to be shown that α_1 cannot have ± 1 as a characteristic root. For this a modification of the preceding development is necessary, because of the manifest singularity of $\sinh \vartheta$. However, if α_1 has ± 1 as a characteristic root, then there is no essential loss of generality in supposing that

$$\alpha_e = \alpha_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Induction yields

$$\begin{pmatrix} \alpha_c & \alpha_c^3 - U_n \\ U_n & \alpha_c \end{pmatrix}^{2^k} = \begin{pmatrix} 1 & 4^k & 0 & 2^{k+1} \\ 0 & 1 & 0 & 0 \\ 2^k & 2^k(4^k-1)/3 & 1 & 4^k \\ 0 & 2^k & 0 & 1 \end{pmatrix},$$

from which we infer

$$Z(2^k) = -T\begin{pmatrix} 2^{-k} & (2^{k+1}+2^{-k})/3 \\ 0 & 2^{-k} \end{pmatrix}T^{-1}\gamma^{-1}.$$

It follows that $Z(m)$ cannot tend to a limit as $m \to \infty$, and the proof is complete.

The criterion becomes considerably simpler if we are interested only in *non-singular* characteristic impedances.

Corollary 14.1. For a passive symmetrical section to admit a non-singular characteristic impedance it is necessary and sufficient that γ be non-singular and that all real characteristic roots of α have absolute value >1. Further, when a non-singular characteristic impedance λ_0 exists, then there is an $n \times n$ matrix ϑ all of whose characteristic roots have positive real parts, such that

$$\chi = \begin{pmatrix} \cosh\vartheta & -(\sinh\vartheta)\lambda_0 \\ -\lambda_0^{-1}\sinh\vartheta & \lambda_0^{-1}(\cosh\vartheta)\lambda_0 \end{pmatrix} = \exp\begin{pmatrix} 0 & -\vartheta\lambda_0 \\ -\lambda_0^{-1}\vartheta & 0 \end{pmatrix}. \tag{21}$$

The matrix ϑ here is called an *attenuation matrix* for the given transmission network N. As justification for this terminology, we observe that when N^m is terminated in a load impedance λ_0 the output vector $i'(m)$ satisfies

$$\begin{cases} e'(m) = \exp(-m\vartheta)e \\ i'(m) = \lambda_0^{-1}\exp(-m\vartheta)\lambda_0 i. \end{cases} \tag{22}$$

Since the real part of each characteristic root of ϑ is positive, we conclude further that $\lim_{m\to\infty} i'(m) = 0$. The output vector is completely attenuated at infinity.

Although the characteristic impedance of N was defined as the limit of the input impedance of N^m terminated in a load admittance $Y_L = 0$, it turns out that for λ_0 non-singular the same limiting behavior holds for a wide variety of terminations.[14]

Theorem 15. Let N be a passive symmetrical section admitting a non-singular characteristic impedance λ_0. If $Z(m)$ is the input impedance for N^m terminated in a load impedance Z_L such that $Z_L + \lambda_0$ is non-singular, then $\lim_{m\to\infty} Z(m) = \lambda_0$.

Proof. From (21) and the termination equation $e'(m) = Z_L i'(m)$ we find

$$Z(m) = [(\cosh m\vartheta)Z_L + (\sinh m\vartheta)\lambda_0][(\sinh m\vartheta)Z_L + (\cosh m\vartheta)\lambda_0]^{-1}\lambda_0,$$

whenever the indicated inverse exists. There follows

$$Z(m) = [U_n + \exp(-m\vartheta)(Z_L - \lambda_0)(Z_L + \lambda_0)^{-1}\exp(-m\vartheta)] \cdot [U_n - \exp(-m\vartheta)(Z_L - \lambda_0)(Z_L + \lambda_0)^{-1}\exp(-m\vartheta)]^{-1}\lambda_0.$$

[14] It, of course, cannot hold for all terminations, since there exist iterative impedances distinct from the characteristic impedance.

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The positivity of the characteristic roots of ϑ then implies that $\exp(-m\vartheta) \to 0$, so that $Z(m)$ exists for large m and tends to λ_0.

A similar discussion shows that $Z(m) \to \lambda_0$ whenever N^m is terminated in a load admittance Y_L such that $Y_L + \lambda_0^{-1}$ is non-singular.

10. CONTINUOUS GROUPS AND TRANSMISSION LINES

The semi-group approach to the theory of transmission networks is particularly well suited to the study of transmission lines. Indeed, from this viewpoint a transmission line appears simply as a one-parameter semi-group of transmission networks (the parameter being the length of the line), and for the analysis of such networks we have an ideal tool in the theory of continuous semi-groups.

The exact postulates for a transmission line are as follows.

Definition 19. By a *transmission line* we mean a semi-group Σ_τ of passive transmission networks $N(\xi)$ defined for all non-negative values of the real parameter ξ and having the properties

(1) for every pair of non-negative numbers ξ_1 and ξ_2

$$N(\xi_1) \cdot N(\xi_2) = N(\xi_1 + \xi_2);$$

(2) for any fixed input vector i the output vector $i'(\xi)$ of $N(\xi)$ is differentiable[15] for all $\xi \geq 0$; and

(3) $N(0)$ is non-singular.

Any network $N(\xi)$ will be called a *section of length* ξ of the transmission line Σ_τ.

These requirements on Σ_τ clearly conform to what is expected of the usual physical transmission lines: (1) the series combination of two sections of length ξ_1 and ξ_2 is a section of length $\xi_1 + \xi_2$; (2) the output vector has a well defined rate of change with respect to length; and (3) the section of length zero (physically the direct connection) is non-singular.

Commutativity of all sections of a transmission line Σ_τ is an obvious consequence of the above definition.

Definition 20. A transmission line Σ_τ will be said to be *symmetrical* provided all sections $N(\xi)$ in Σ_τ possess section symmetry.

Although in the physical case it is customary to deal only with symmetrical transmission lines, many of the important results apply equally well to non-symmetrical lines. We shall discuss symmetrical lines later, placing particular emphasis on the exact role played by the requirement of section symmetry.

The transmission matrix for $N(\xi)$ will be denoted by $\chi(\xi)$, so that $i'(\xi) = \chi(\xi)i$. From (1) it is evident that

$$\chi(\xi_1)\chi(\xi_2) = \chi(\xi_1 + \xi_2),$$

[15] The derivative of a vector (or matrix) function of a scalar is the vector (or matrix) obtained by taking the derivatives of the respective components.

and taking account of the non-singularity of $\chi(0)$, we then obtain

$$\chi(0) = U_{2n}.$$

Furthermore, the differentiability condition (2) implies that $\chi(\xi)$ is differentiable for all $\xi \geq 0$.[16]

According to the theory of continuous groups,[17] every continuous group of differentiable functions satisfies a certain set of ordinary differential equations, called the *fundamental differential equations* of the group. The same reasoning applies to semi-groups, and for Σ_r we proceed as follows. From the expression $N(\eta - \xi) \cdot N(\xi) = N(\eta)$, where $\eta \geq \xi$, we obtain

$$\chi(\eta - \xi)t'(\xi) = t'(\eta),$$

and a partial differentiation with respect to ξ yields

$$\chi(\eta - \xi)\frac{dt'(\xi)}{d\xi} - \frac{d\chi(\eta - \xi)}{d(\eta - \xi)}t'(\xi) = 0.$$

Setting $\eta = \xi$ reduces this to

$$\frac{dt'(\xi)}{d\xi} = \omega t'(\xi), \tag{23}$$

where

$$\omega = \left.\frac{d\chi(\xi)}{d\xi}\right|_{\xi=0}. \tag{24}$$

Equation 23, the fundamental differential equation for Σ_r, holds for the input vector t fixed at any arbitrary value. We shall refer to (23) as the *transmission line equation*, and to ω as the *generating matrix* for the transmission line Σ_r.

By differentiating $t'(\xi) = \chi(\xi)t$ and using the arbitrariness of t, we find that (23) is equivalent to the matrix differential equation

$$\frac{d\chi(\xi)}{d\xi} = \omega\chi(\xi). \tag{25}$$

In conjunction with the initial condition $\chi(0) = U_{2n}$, Eq. 25 has the solution $\chi(\xi) = \exp(\xi\omega)$, and uniqueness of this solution is assured by the usual uniqueness theorems for systems of ordinary differential equations.

Theorem 16. If ω is the generating matrix for a transmission line Σ_r, then every section $N(\xi)$ of Σ_r has transmission matrix

$$\chi(\xi) = \exp(\xi\omega).$$

Conversely, given any $2n$-dimensional matrix ω, there corresponds a unique transmission line with generating matrix ω.

[16] For t the unit vector along the kth axis, we note that $t_j'(\xi) = \chi_{jk}(\xi)$.

[17] A systematic treatment of the relevant aspects of this theory can be found in 2.

It follows that every section of a transmission line is non-singular, and we have, in fact,

Corollary 16.1. Every transmission line Σ_τ can be extended from a semi-group to a group Γ_τ by defining its sections $N(\xi)$ for *all* real ξ as the passive transmission networks with matrices

$$\chi(\xi) = \exp(\xi\omega),$$

where ω is the generating matrix of Σ_τ.

Thus, there is no loss of generality in supposing that a given transmission line is actually a *group* of passive sections $N(\xi)$ defined for all real ξ and satisfying (1) and (2) of definition 19 for all real ξ, ξ_1, ξ_2.

According to the Lie theory of continuous groups, the fundamental differential Eq. 23 is considered as defining a trajectory of the group Γ_τ, since it gives rise to the finite transformations $t'(\xi) = \chi(\xi)t$. As ξ varies, $t'(\xi)$ moves along this trajectory from the initial point $t'(0) = t$. It is important in this theory to examine the so-called "infinitesimal transformations" of the group, corresponding to motion along the tangent to the trajectory at any point ξ, rather than along the trajectory itself. Since, by (24) and theorem 16, a transmission line is completely determined by its behavior in the neighborhood of $\xi = 0$, it suffices to confine our attention to the infinitesimal transformations at the origin:

$$t'(\Delta\xi) = t + \left.\frac{dt'(\xi)}{d\xi}\right|_{\xi=0} \Delta\xi = (U_{2n} + \omega\Delta\xi)t.$$

In a completely intuitive manner we shall investigate the "infinitesimal networks" $N(\Delta\xi)$ with transmission matrices $\chi(\Delta\xi) = U_{2n} + \omega\Delta\xi$, neglecting higher order terms in $\Delta\xi$ wherever they arise. This will serve as a sort of divining rod, to unearth properties of Γ_τ which can then be verified rigorously, but which would otherwise appear somewhat unmotivated. Furthermore, we have at our disposal the theorem (2, p. 63): any invariant of the infinitesimal networks is actually an invariant of the finite networks.

Writing ω in $n \times n$ block form as

$$\omega = \begin{pmatrix} u & z \\ y & v \end{pmatrix}, \tag{26}$$

we obtain

$$\chi(\Delta\xi) = \begin{pmatrix} U_n + u\Delta\xi & z\Delta\xi \\ y\Delta\xi & U_n + v\Delta\xi \end{pmatrix}. \tag{27}$$

As an example of the utility of the notion of infinitesimal network, let us consider the property of impedance symmetry. From (*a*), (*c*), (*d*) of theorem 3 it is immediate that $N(\Delta\xi)$ is impedance-symmetric if and only if $y^* = y$, $z^* = z$, and $v^* = -u$. We then verify that this criterion remains valid for the finite networks.

Theorem 17. Let Γ_τ be a transmission line with generating matrix (26). Then a necessary and sufficient condition for $N(\xi)$, $\xi > 0$, to be impedance-symmetric in the neighborhood of the origin is that $y^* = y$, $z^* = z$, and $v^* = -u$; or, equivalently that $S\omega^*S = \omega$, where

$$S = \begin{pmatrix} 0 & -U_n \\ U_n & 0 \end{pmatrix}.$$

Moreover, when this condition is satisfied, all $N(\xi)$ in Γ_τ are impedance-symmetric.

Proof. A direct calculation shows the equations $y^* = y$, $z^* = z$, $v^* = -u$ to be equivalent to $S\omega^*S = \omega$, and this is clearly equivalent to $S^{-1}\omega^*S = -\omega$. If ω satisfies this condition, then for all ξ we have

$$S^{-1}\chi^*(\xi)S = S^{-1}[\exp(\xi\omega)]^*S = S^{-1}\exp(\xi\omega^*)S = \exp(\xi S^{-1}\omega^*S)$$
$$= \exp(-\xi\omega) = \chi^{-1}(\xi).$$

Hence, by (9′), which is equivalent to (9), the condition $S\omega^*S = \omega$ is *sufficient* for impedance-symmetry of all $N(\xi)$.

For the *necessity* we start with $\chi^*(\xi)S\chi(\xi) = S$ for $0 \leq \xi < \xi_0$ and obtain by differentiation

$$\frac{d\chi^*(\xi)}{d\xi} S\chi(\xi) + \chi^*(\xi)S\frac{d\chi(\xi)}{d\xi} = 0.$$

Setting $\xi = 0$ yields $\omega^*S + S\omega = 0$, and there follows $S\omega^*S = \omega$.

Calculation of the iterative impedances gives rise to a further application of infinitesimal networks. In fact, the iterative impedances of $N(\Delta\xi)$ are easily seen (theorem 8) to be the $n \times n$ matrices λ satisfying

$$\lambda y\lambda + \lambda v - u\lambda - z = 0, \tag{28}$$

and we proceed to show that these are exactly the iterative impedances of Γ_τ (in the following sense).

Definition 21. By an *iterative impedance* of a transmission line Γ_τ we mean an $n \times n$ matrix which is an iterative impedance of every section $N(\xi)$ of Γ_τ.

Theorem 18. Let Γ_τ be a transmission line with generating matrix (26). Then a necessary and sufficient condition for $N(\xi)$, $\xi > 0$, to have λ as an iterative impedance in the neighborhood of the origin is that λ be an iterative impedance of the transmission network $t' = \omega t$, that is, that λ satisfy (28). Moreover, when this condition is satisfied, λ is an iterative impedance of Γ_τ.

Proof. If λ satisfies (28), then

$$\Lambda^{-1}\omega\Lambda = \begin{pmatrix} * & 0 \\ * & * \end{pmatrix},$$

where Λ is defined as

$$\Lambda = \begin{pmatrix} U_n & \lambda \\ 0 & U_n \end{pmatrix}.$$

It follows without difficulty that

$$\Lambda^{-1}\chi(\xi)\Lambda = \exp(\xi\Lambda^{-1}\omega\Lambda) = \begin{pmatrix} * & 0 \\ * & * \end{pmatrix},$$

so that

$$\lambda\gamma(\xi)\lambda + \lambda\delta(\xi) - \alpha(\xi)\lambda - \beta(\xi) = 0. \tag{29}$$

Hence, (28) is *sufficient* for λ to be an iterative impedance of Γ_τ. For the *necessity* we merely evaluate the derivative of (29) at $\xi = 0$.

A correlation between transmission lines and general transmission networks is afforded by the following observation.

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Theorem 19. If N is any non-singular transmission network with transmission matrix χ, then there exists a transmission line Γ_τ with generating matrix $\omega = \log \chi$ such that N is the section of Γ_τ having unit length.

Clearly, $\log \chi$ exists, since χ is non-singular. It is obvious also that the transmission line Γ_τ is not unique, in view of the fact that ω can be taken as any determination of $\log \chi$.

11. SYMMETRICAL TRANSMISSION LINES

We turn now to the usual physical case, in which every section is symmetrical (that is, unaffected by an interchange of corresponding input and output terminals).

The first two of the section symmetry conditions of (14) lead, upon differentiation, to

$$\alpha(\xi)\frac{d\alpha(\xi)}{d\xi} + \frac{d\alpha(\xi)}{d\xi}\alpha(\xi) = \beta(\xi)\frac{d\gamma(\xi)}{d\xi} + \frac{d\beta(\xi)}{d\xi}\gamma(\xi)$$

and

$$\delta(\xi)\frac{d\delta(\xi)}{d\xi} + \frac{d\delta(\xi)}{d\xi}\delta(\xi) = \gamma(\xi)\frac{d\beta(\xi)}{d\xi} + \frac{d\gamma(\xi)}{d\xi}\beta(\xi),$$

and thence to

$$u = \left.\frac{d\alpha(\xi)}{d\xi}\right|_{\xi=0} = 0$$

and

$$v = \left.\frac{d\delta(\xi)}{d\xi}\right|_{\xi=0} = 0.$$

Theorem 20. A necessary and sufficient condition that a transmission line Γ_τ be symmetrical is that its generating matrix have the $n \times n$ block form

$$\omega = \begin{pmatrix} 0 & z \\ y & 0 \end{pmatrix}.$$

Proof. Only the sufficiency remains to be established. Taking ω in the above block form and J as specified in definition 14, we readily verify that $J^{-1} = J$ and $J^{-1}\omega J = -\omega$. The transmission matrix $\chi(\xi)$ of any $N(\xi)$ in Γ_τ thereby satisfies

$$\begin{aligned}[J\chi(\xi)]^2 &= J^{-1}\exp(\xi\omega)J\exp(\xi\omega) \\ &= \exp(\xi J^{-1}\omega J)\exp(\xi\omega) \\ &= \exp(-\xi\omega)\exp(\xi\omega) = U_{2n}.\end{aligned}$$

Hence, by theorem 5, $N(\xi)$ must be symmetrical.

Equation 23 now splits into a pair of equations, which we recognize as the classical transmission line equations:

$$\left.\begin{aligned}\frac{de'(\xi)}{d\xi} &= zi'(\xi) \\ \frac{di'(\xi)}{d\xi} &= ye'(\xi)\end{aligned}\right\}. \tag{30}$$

Physically, z can be considered as the impedance per unit length of the transmission line obtained by setting $y = 0$, and y can be considered as the admittance per unit length of the transmission line obtained by setting $z = 0$. This observation is the basis of a heuristic derivation of Eqs. 30 prevalent in elementary textbooks on transmission line theory. However, a valid reduction of (23) to (30) depends strongly on section symmetry.

A number of properties of symmetrical transmission lines follow at once from those for general lines. Also, in the light of theorem 19 these results apply to any non-singular symmetrical transmission section, and they provide an alternative approach to the problems discussed in sections 7 and 9.

Theorem 21. Let Γ_r be a symmetrical transmission line with generating matrix

$$\omega = \begin{pmatrix} 0 & z \\ y & 0 \end{pmatrix}.$$

Then

(1) the sections of Γ_r are impedance-symmetric if and only if y and z are symmetric matrices;
(2) the iterative impedances of Γ_r are the $n \times n$ matrices λ satisfying $\lambda y \lambda = z$.

Proof. Corollaries of theorems 17 and 18.

Thus, when an iterative impedance λ exists, we have $(\lambda y)^2 = zy$.

Corollary 21.1. If Γ_r is a symmetrical transmission line having λ as an iterative impedance, then zy must admit a square root and λ must satisfy

$$\lambda y = \sqrt{zy}$$

for some determination of the square root.

It follows that λ can be found explicitly as $\lambda = \sqrt{zy}\,y^{-1}$ provided y is non-singular, and that iterative impedances always exist whenever both y and z are non-singular.

Series expansions for the submatrices of $\chi(\xi)$ are found from

$$\begin{pmatrix} \alpha(\xi) & \beta(\xi) \\ \gamma(\xi) & \delta(\xi) \end{pmatrix} = \exp \begin{pmatrix} 0 & \xi z \\ \xi y & 0 \end{pmatrix}$$

as

$$\left.\begin{aligned} \alpha(\xi) &= U_n + (\xi^2/2!)zy + (\xi^4/4!)(zy)^2 + \cdots \\ \beta(\xi) &= z[\xi + (\xi^3/3!)yz + (\xi^5/5!)(yz)^2 + \cdots] \\ \gamma(\xi) &= y[\xi + (\xi^3/3!)zy + (\xi^5/5!)(zy)^2 + \cdots] \\ \delta(\xi) &= U_n + (\xi^2/2!)yz + (\xi^4/4!)(yz)^2 + \cdots, \end{aligned}\right\} \tag{31}$$

which converge for all ξ, y, z. There follows

Theorem 22. Let Γ_r be a symmetrical transmission line with generating matrix

$$\omega = \begin{pmatrix} 0 & z \\ y & 0 \end{pmatrix}.$$

If Γ_τ admits a non-singular iterative impedance λ, then the transmission matrices of the sections $N(\xi)$ of Γ_τ have the form

$$\chi(\xi) = \begin{pmatrix} \cosh \xi\varphi & -(\sinh \xi\varphi)\lambda \\ -\lambda^{-1} \sinh \xi\varphi & \lambda^{-1}(\cosh \xi\varphi)\lambda \end{pmatrix},$$

where $\varphi = -\lambda y$.

As in (22), terminating $N(\xi)$ in a load impedance λ then yields the relations

$$\left.\begin{aligned} e'(\xi) &= \exp(-\xi\varphi)e \\ i'(\xi) &= \lambda^{-1} \exp(-\xi\varphi)\lambda i. \end{aligned}\right\} \tag{32}$$

For a transmission line Γ_τ we distinguish between the *forward characteristic impedance* and the *backward characteristic impedance*, which are defined in terms of the input impedance $Z(\xi)$ of $N(\xi)$ as

$$\vec{\lambda} = \lim_{\xi \to \infty} Z(\xi)$$

and

$$\overleftarrow{\lambda} = \lim_{\xi \to -\infty} Z(\xi),$$

respectively, whenever the indicated limits exist. It is easily seen from (31) that a symmetrical transmission line admits a forward characteristic impedance $\vec{\lambda}$ if and only if it admits a backward characteristic impedance $\overleftarrow{\lambda}$, and we then have $\overleftarrow{\lambda} = -\vec{\lambda}$. Furthermore, it is obvious that if a transmission line admits a forward characteristic impedance $\vec{\lambda}$, then every section of positive length has characteristic impedance $\vec{\lambda}$.

We omit the lengthy discussion of conditions for the existence of a forward characteristic impedance in the general case, since this discussion parallels closely that given for the proof of theorem 14. However, for the case of non-singular forward characteristic impedances we have

Theorem 23. Let Γ_τ be a symmetrical transmission line with generating matrix

$$\omega = \begin{pmatrix} 0 & z \\ y & 0 \end{pmatrix}.$$

A necessary and sufficient condition that Γ_τ admit a non-singular forward characteristic impedance $\vec{\lambda}$ is that all real characteristic roots of zy be positive. Then

$$\vec{\lambda} = -\sqrt{zyy^{-1}},$$

where the indicated square root is that for which all characteristic roots have positive real parts.

Proof. If Γ_τ has a non-singular forward characteristic impedance $\vec{\lambda}$, then $\vec{\lambda}$ is an iterative impedance, and

$$Z(\xi) = -[\gamma(\xi)]^{-1}\vartheta(\xi) = (\coth \xi\varphi)\vec{\lambda}.$$

Hence, $\coth \xi\varphi$ must approach U_n as $\xi \to \infty$, and this requires that all characteristic roots of φ have positive real parts. Since (by corollary 21.1) φ is some square root of zy, it follows that all real characteristic roots of zy must be positive.

Conversely, if all real characteristic roots of zy are positive, then we can choose $\varphi = \sqrt{zy}$ so that all characteristic roots of φ have positive real parts. Then for $\lambda = -\varphi y^{-1}$ there results $Z(\xi) = (\coth \xi\varphi)\lambda \to \lambda$, proving that $\vec{\lambda} = \lambda = -\sqrt{zy}y^{-1}$.

By virtue of (32) the $n \times n$ matrix $\varphi = -\vec{\lambda}y$ determines the propagation of signals along an infinite line, and it is appropriately called the *propagation matrix* of the line.

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[11] References 1 and 3 are included as general sources of background material in modern algebra and in the theory of networks, but no explicit reference to them appears in the text.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

VISCOSIMETRY OF GASES AT HIGH TEMPERATURES AND PRESSURES

To provide data needed for the design of gas turbines and jet engines, the National Bureau of Standards is conducting a broad program of research on the viscosity of gases at elevated temperatures and pressures. Under the sponsorship of the National Advisory Committee for Aeronautics, hydrocarbon mixtures and combustion products composed of various proportions of nitrogen, carbon dioxide, argon, oxygen, carbon monoxide, and water vapor are being studied. In the course of this work, J. C. Westmoreland of the NBS staff has developed a unique type of flowmeter[1] which accurately controls and measures the flow of very small volumes of gas, such as those required in a small capillary tube viscosimeter. As a result of this development, it has been possible to obtain data hitherto unavailable on the viscosity of pure gases and their mixtures.

Accurate data on the viscosity of gases are required in designing transmission and measurement systems for fuel gases and in constructing gas-flow systems in chemical plants. Because the viscosity of a gas is an important factor in the rate of heat transmission between gases and metals, gas-viscosity data are also needed in the design of gas turbines and jet engines. With the continued development of new engines of this kind, the need for information on the viscosity of the exhaust gases at high temperatures is becoming increasingly important.

At present, available data on the viscosity of gases are very meager, particularly at temperatures above 1000° F. To meet this need, the National Bureau of Standards has undertaken to determine the viscosity of pure gases and gas mixtures at pressures from 5 to 10,000 psi. and over the temperature range from 60° to 2500° F.

In the NBS program, viscosities are measured by timing the flow of the gas under study through a very fine capillary tube. If the dimensions of the tube are known, the viscosity can be computed by means of Poiseuille's law[2] from the volume rate of flow through the tube and the pressure drop from inlet to outlet of the tube. The capillary tubes

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* Communicated by the Director.

[1] For further technical details, see "A Meter for Timing the Flow of Very Small Volumes of a Gas," by J. C. Westmoreland, NACA Research Memorandum RM52109.

[2] According to Poiseuille's law, when a fluid flows through a capillary tube, the volume rate of flow is given by $\frac{\pi r^4 \Delta p}{8\eta l}$, where r is the radius and l the length of the capillary, Δp is the pressure drop across the tube, and η is the coefficient of viscosity of the fluid.

are of Inconel, with average internal bores of 0.038 cm. and lengths up to 434 cm. For high-pressure operation the tubes are enclosed so that they have only to withstand the small pressure differential caused by the flow of gas. The capillary enclosure is heated electrically, and temperatures are measured with chromel-alumel thermocouples. At high temperatures a coiled capillary is used.

In order to obtain as accurate viscosity values as possible, it was decided to measure the rate of flow of the gas through the capillaries by means of some metering device employing a volumetric displacement. This method would have the advantage that the volume could be evaluated directly by conventional methods and would not involve the measurement of an auxiliary quantity. As no flowmeter capable of measuring the extremely small flow rates through the tubes was available, it was necessary to design such an instrument. The apparatus that was developed not only determines these flow rates accurately at pressures up to 500 atmospheres but also includes a means of regulating the rate of flow and of measuring or pre-establishing the pressure drop over the capillary.

In previous investigations, determination of the viscosity of a gas at high pressures has required a knowledge of the compressibility effects of the particular gas being tested. However, in the NBS experiments the new flowmeter is used to measure the volume rate of gas flow at the static pressure of the test, and an absolute knowledge of the compressibility of the gas may not be required. Under the conditions of operation of this device, the Poiseuille expression is completely defined at the test pressure and temperature, and the relationship is essentially independent of the thermodynamic deviations from an ideal gas.[3]

The flowmeter developed at the Bureau consists essentially of a calibrated volume chamber, a float-valve chamber, and a gas-mercury reservoir. These are connected in with the calibrated capillary tube to form a closed system in which the gas under study flows out of the reservoir and through the capillary tube to displace mercury in the volume chamber. The float valve regulates the flow of mercury out of the volume chamber as it is displaced by the incoming gas, and at the same time maintains constant the height of a mercury column which provides a hydrostatic pressure causing the gas to flow through the capillary tube. As the mercury passes out of the volume chamber, it breaks electrical contact with successive platinum electrodes, thus activating relays which start and stop a clock. In this way the time required for the measured volume of gas to flow through the capillary tube is obtained.

[3] The compressibility of a gas is not independent of temperature; and if the flowmeter cannot be operated at the same temperature and pressure as the capillary tube, it will, of course, be necessary to make the appropriate correction to the volume.

No energy from the gas stream is employed in replacing the mercury, which is made to flow by gravity out of the calibrated container through the float valve chamber at a rate equal to the rate of gas inflow. The replaced mercury is then added to the top of the mercury in the hydrostatic column, replacing mercury which passes out of the small-diameter tube forming the power part of the column and into the gas-mercury reservoir. The mercury which thus enters the reservoir in turn replaces the gas which is continually leaving the reservoir to pass through the calibrated capillary tube. A pressure-equalization tube which joints the calibrated gas collection chamber to the float valve chamber insures that the pressure differential across the capillary tube shall be only that due to the column of mercury above the gas-mercury reservoir.

The volume chamber is made of glass and is divided into two equal parts, one above the other, with a necked section in between the two and at the base of the lower chamber. The principal purpose of the upper volume is to provide a starting period during which flow conditions may become stabilized. The necked sections make the cross-sectional area small in the plane of the contact break.

The float chamber, float, and valve are all made of methyl methacrylate. It was found that a valve and valve seat of this plastic material are not eroded by the flowing mercury as a steel valve would be. Moreover, the mercury remains clean in contact with the methyl methacrylate.

In the operation of the flowmeter, the entire apparatus, with all of the mercury in the reservoir, is first connected to a high-vacuum pump and purged of all gas as completely as possible. A valve in the column between the float-valve chamber and the gas-mercury reservoir is then closed, and a gas-sample inlet valve is opened, admitting gas from an external container to the reservoir. This causes the mercury in the reservoir to rise through an auxiliary connecting tube into the passage above the float valve. As the tube below the float valve is now blocked off, the mercury will first fill the float chamber enough to close the float valve and will then fill the calibrated volume chamber. A cock in the auxiliary connecting tube is then closed. When the inflow of the sample gas has brought the pressure within the reservoir to the desired value, the gas-sample inlet valve is also closed.

Before measurements are taken, sufficient time must elapse for pressure conditions to equalize throughout the system. Then the meter is placed in operation by opening the valve in the column connecting the float-valve chamber with the gas-mercury reservoir. The outflow of mercury from the float-valve chamber lowers the float, opening the float valve so as to admit mercury from the calibrated volume container into the float-valve chamber. Whenever this inflow from the volume chamber exceeds the outflow into the reservoir,

the float valve automatically closes. Thus the pressure which the mercury column exerts on the gas entering the capillary tube is kept constant, and the rate at which mercury flows out of the volume chamber is controlled by the rate of flow of gas through the capillary.

In making the determinations, the apparatus is first pressurized at a specific temperature. Flow measurements are then carried out at decreasing pressure levels, and the viscosities are computed for each value of pressure. This procedure gives a family of isotherms showing the variation in the viscosity of the gas sample with pressure at the temperature of the test.

In addition to its use in viscosity studies, the new flowmeter may find application in determining the specific gravity of a gas by the effusion method. For this purpose the capillary tube would be replaced by a thin metal diaphragm containing a small orifice.

Because the viscosity of a gas changes rapidly with temperature, the Bureau's flowmeter should also prove useful for measuring temperatures, particularly at elevated levels where thermoelectric means have proved unreliable. For such use, the capillary tube would be coiled into a compact sensing unit which would be charged once and for all with a suitable gas. The time interval for a given volume of the gas to flow through the capillary would then be a measure of the average temperature of the capillary.

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UNDERGROUND CORROSION OF GALVANIZED STEEL

A study of underground corrosion of hot-dipped galvanized steel pipe has recently been completed by the National Bureau of Standards. Results confirm previous NBS work in showing that galvanized steel having 3 ounces of zinc per square foot of exposed surface is highly resistant to corrosion in many soils which are very corrosive to bare steel. Conducted by Irving A. Denison and Melvin Romanoff of the Bureau's corrosion laboratory, the present study was an outgrowth of earlier NBS exposure tests of zinc coatings for underground use.

For the Bureau's study, short lengths of both galvanized and uncoated steel pipe, and also plates of zinc, were buried at 15 test sites for periods up to 13 years. Located in widely separated parts of the United States, the test sites represented a wide range of soil properties. After each of five periods of exposure, a set of specimens of each material was removed and returned to the NBS laboratories. After removal of the corrosion products, determinations were made of loss in weight, depth of the deepest pits, and the percentage of area of the galvanized specimens on which coating remained.

Although the nominal weight of the zinc on all the coated specimens was 3 ounces per square foot, the actual thickness varied over a wide range. This was shown by a large number of thickness measurements

made by a magnetic method, using unexposed samples of pipe from the same lot as the buried specimens. Additional determinations by an electrolytic stripping method showed that a large part of the zinc that was applied to the steel pipe was converted into alloys of zinc and iron.

The zinc coatings provided good protection in most of the soils. In one soil in which bare steel pipe was perforated by corrosion after exposure for only a few years, the coating on the galvanized specimens remained perfectly continuous throughout the entire 13-year period. In only two of the 15 soils, both organic, was the zinc coating of negligible protective value.

An interesting finding of the NBS study was the high corrosion resistance that the galvanized specimens continued to show in most of the soils after the outer zinc coating, and even after the zinc-iron alloy layer, had entirely corroded away. This continuing protection is tentatively attributed to an inorganic coating, probably silicious, believed to have been deposited by galvanic action between the outer zinc coating and the underlying steel or alloy layer. This tentative explanation is based in part on unpublished studies which indicate that the zinc-iron alloy layer does not protect steel sacrificially (cathodically), and that the alloy is no more resistant than steel to soil corrosion.

Analysis of the data obtained in the course of the two NBS field studies shows that the minimum weight of zinc coating required to protect steel for a minimum of 10 years depends on the nature of the soil environment. In the NBS studies a 2-ounce coating gave sufficient protection in inorganic oxidizing soils, but for inorganic moderately reducing soils a 3-ounce coating was required. Highly reducing soils, both organic and inorganic, require coatings heavier than 3 ounces per square foot.

In order to obtain maximum life from galvanized pipe in practice, it is necessary either to construct the entire piping system of galvanized pipe or else to electrically insulate the galvanized sections from pipe of other metals. Otherwise, the zinc coating will be removed by galvanic action.

NOTE: Further details of this NBS investigation are given in "Corrosion of Galvanized Steel in Soils," by Irving A. Denison and Melvin Romanoff, *J. Research NBS*, Vol. 49, p. 299 (Nov. 1952) (NBS Research Paper 2366; 10 cents from Superintendent of Documents, Government Printing office, Washington 25, D.C.) For information on earlier related NBS work, see "Corrosion of Metals Underground," NBS *Tech. News Bull.*, Vol. 34, p. 48 (April 1950).

THE FRANKLIN INSTITUTE

MINUTES OF STATED MEETING

April 15, 1953

The Stated Monthly Meeting of The Franklin Institute was held on April 15, 1953, in the Lecture Hall. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. Approximately 220 persons attended.

The President asked for approval of the Minutes of the Stated Meeting in February which were printed in the March JOURNAL. There being no corrections or additions, they were approved as printed. He stated that the Minutes of the Stated Meeting in March were being printed in the April JOURNAL, which had not yet been mailed to members. Approval, therefore, was postponed until the next meeting.

The Assistant Secretary, reporting for the Secretary, announced the following elections to membership for the month of March:

Active	26
Associate	31
Student	7

and a total membership of 6,632 as of March 31, 1953.

He spoke of the dramatic and spectacular "Rocket to Mars," which will be shown in the Planetarium during April and May, and to the annual "Science Fair," sponsored by *The Philadelphia Inquirer* and The Franklin Institute, which will be held in Franklin Hall from April 18th to April 26th, inclusive. The purpose of the "Science Fair" is to promote interest in science and the crafts among students in the Philadelphia area.

The President drew attention to the privileges of Student Membership in the Institute. He urged members to encourage their young friends and relatives to become Student Members.

The President then introduced the speaker for the evening—Lieutenant General Leslie R. Groves (United States Army, Retired), Vice-President in charge of Scientific Research at Remington Rand, Inc. General Groves' interesting talk included the objectives of industrial research, its role in industry, and financing an industrial research program. Among the examples given from his own experience in governmental and in industrial research was "The Atom vs. the Univac."

The meeting adjourned at 9:35 P.M. with a rising vote of thanks to General Groves.

JOHN FRAZER,
Assistant Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, April 8, 1953.)

HALL OF THE COMMITTEE,
PHILADELPHIA, APRIL 8, 1953.

COLONEL HIRAM B. ELY in the Chair.

The following reports were presented for final action:

No. 3263: Henderson Medal.

This report recommended the award of the George R. Henderson Medal to Adolph Meyer, of Zurich, Switzerland, "In consideration of his basic contributions to scientific research on the Gas Turbine power plant and, in particular, to his pioneer work in the development of the first successful Gas Turbine Locomotive."

No. 3264: Clamer Medal.

This report recommended the award of the Francis J. Clamer Medal to Robert Franklin Mehl, of Pittsburgh, Pennsylvania, "In recognition of his numerous useful contributions in the fields of theoretical and applied metallography and metallurgy."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS

April 15, 1953

ACTIVE FAMILY

Arthur Borowsky
Martin M. Decker
Philip M. Lessig, D.O.
Girard Seitter
John H. Simmons

ACTIVE

A. M. Ball
Ralph H. Beter
Harry Coren
Alfred Dallago
Roy J. A. Erickson
E. Jay Ferry
Norman Finkelstein
Robert Z. Gondos
Christian M. Gottschau
Baldwin M. Haines
Bevier Hasbrouck
Max Hecht
George N. Johnson
Major Robert B. Kadel
Robert H. Stern
Howard J. Strauss
Elmer C. Swett
Charles Hinchman Topping
Charles J. Webb

NECROLOGY

Charles E. Beury, LL.D. '36
Cameron Burnside '52
Albert O. H. Grier '38
Philip H. Rosenbach '36

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A. M. until 5 P. M.; Wednesdays and Thursdays from 2 P. M. until 10 P. M.

On and after June 1, the Library hours will be 9 A. M. until 5 P. M. on Mondays, Tuesdays, Thursdays and Fridays; 2 P. M. until 10 P. M. on Wednesdays; and 9 A. M. until 12 noon on Saturdays.

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MOON, PARRY AND DOMINA EBERLE SPENCER: Some Coordinate Systems Associated with Elliptic Functions.
SWANN, W. F. G.: Fundamentals in the Behavior of Electrets.
LAUER, HENRI: Operating Modes of a Servomechanism with Non-Linear Friction.
COULSON, THOMAS: Early Research at The Franklin Institute.
COURT, ARNOLD: Wind Extremes as Design Factors.
ALDRICH, LELAND: Solution of Algebraic Equations.
MINORSKY, N.: On Interaction of Nonlinear Oscillators.
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SWANN, W. F. G.: The Case of Charged Density Distribution *versus* Semi-Permanent Polarization as a Basis for Electret Behavior.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

THE DIAZO-COUPLING REACTION FOR THYMINE

BY

EUGENE D. DAY AND WILLIAM A. MOSHER

Thymine or 2,6-dihydroxy-5-methylpyrimidine belongs to the class of 5-substituted pyrimidines, the diazo-coupled derivatives of which have been characterized as extremely unstable compounds that defy isolation. Pyrimidines which are not substituted in the 5-position have been shown by Lythgoe *et al* (1) to couple with diazonium salts to form 5-azopyrimidine derivatives which can be easily isolated and identified. As early as 1908 Johnson and Clapp (2) tried without success to crystallize the red product formed by the reaction between thymine and *p*-diazobenzene sulfonic acid in sodium hydroxide solution. In 1944 Lythgoe *et al* (1) coupled another 5-substituted pyrimidine, 2,6-dihydroxy-4-methyl-5-*n*-butyl-pyrimidine, with diazotized *p*-chloroaniline. Their endeavors to isolate this red product failed.

During the development of an improved procedure for the colorimetric estimation of thymine (3), stability of the red azo thymine derivative in solution was attained through the use of the glycerol-alkali reagent. This red compound was crystallized and shown to be a pH-sensitive product. The rates of coupling and of color development indicated that the coupling reaction for thymine is oxygen-catalyzed and the production of red color is pH-dependent.

EXPERIMENTAL

The Coupling Reaction for Thymine. A solution containing 0.1800 mg. purified thymine/ml. was mixed with twice its volume of 1.2 per cent sodium carbonate. 6.0-ml. aliquot portions of this solution were added to oxygen-containing Klett tubes according to the procedure of Day and Mosher (3). Coupling times between additions of diazo reagent and glycerol-alkali were varied on one-minute intervals from 1 to 25 min. The maximum optical density, in terms of Klett colorimeter readings, of red color produced in each tube was then obtained. The optical density was converted by means of the standard equation for thymine previously described (3) first into the fraction of the original concentration of 0.1800 mg. thymine/ml. which had coupled and then into micromoles of coupled thymine/ml. present in the final red-colored solution.

When the amount of coupled thymine formed was plotted against

the time of coupling, a straight line was produced (5) which showed that the coupling reaction for thymine was a zero-order reaction. Moreover, as soon as an optical density was reached which corresponded to 0.1800 mg. thymine/ml. it remained constant when the coupling time was extended.

Rate of Formation of Red Color from Coupled Thymine. A solution containing 0.1111 mg. purified thymine/ml. was mixed with twice its volume of 1.2 per cent sodium carbonate. 6.0 ml. of this solution were then added to an oxygen-containing Klett tube. 2.0 ml. of the diazo reagent were added and coupling was permitted to take place for 20 min. at 30° C. As soon as 2.0 ml. of the glycerol-alkali reagent were added and thoroughly mixed with the coupled thymine solution, a colorimeter reading was taken approximately every minute until a maximum optical density of red color was reached.

Each colorimeter reading was translated into mg. thymine/ml. by means of the standard equation, subtracted from 0.1111 mg. thymine/ml. in order to give the amount of coupled thymine not yet converted into the red form, and then translated into the number of micromoles of unconverted coupled thymine in the final solution. When the logarithm of each final value was plotted against the time of color development, a straight line was produced (5) which clearly indicated that the reaction involved in the formation of red color was of the first order.

Isolation of the Red-Colored Complex. Two hundred ml. of a solution containing 0.2 mg. thymine/ml. were mixed with 400 ml. of 1.2 per cent sodium carbonate solution, and the mixture was added to an oxygen-containing 1000-ml. volumetric flask. Two hundred ml. of the diazo reagent were then added, the solution thoroughly mixed, and the flask and its contents kept at 30° C. in a water bath for 20 min. Two hundred ml. of a freshly prepared solution containing one part glycerol and one part 6 *N* sodium hydroxide were added and the solution thoroughly mixed by shaking. After 1 hr., by which time the red color had reached a maximum optical density, the solution was mixed with an equal volume of pyridine to give a homogeneous mixture. Sodium hydroxide pellets were added slowly until the less basic pyridine separated from the glycerol-alkali-water phase. The pyridine phase which contained a small amount of water, was combined with 200 ml. ethyl alcohol and 800 ml. benzene and thoroughly shaken. After a few minutes about 20 ml. of a red-colored alcohol-water phase separated from the pyridine-benzene-alcohol emulsion. The 20 ml. were then evaporated in air by frequent additions first of 100 per cent ethyl alcohol and then anhydrous ether. A small amount of yellow crystals was finally obtained.

When these yellow crystals were dissolved in water or in sodium carbonate solution, a yellow color resulted, but when they were dis-

solved in 1 *N* sodium hydroxide, a red color appeared slowly and reached a fairly high intensity. Moreover, when sodium hydroxide was added either to the yellow carbonate or the water solution of the crystals, a red color also slowly appeared.

DISCUSSION

When the phenomenon of a zero-order reaction for the coupling of thymine is considered together with the observation (5) that the coupling of thymine occurs under nitrogen only to a very limited extent, under air very slowly but to completion, and under oxygen rapidly to completion, it can be concluded that the coupling reaction for thymine involves an oxygen-catalyzed process which is the rate-determining reaction.

When the phenomenon of a first order reaction for the development of red color in solutions of coupled thymine is considered together with the observation that the color of the isolated crystalline product in solution is pH-dependent, it can be concluded that the process of red color development involves an equilibrium between the red and yellow forms, the former of which is predominant at a higher alkaline pH.

The mechanism proposed by Hunter to explain the diazo-coupling and color-development reactions for thymine (4) can now be seen to be too complex to fit the facts observed. In that mechanism thymine couples with the diazo reagent in carbonate to form a yellow oxy-azo compound which, in sodium hydroxide, rearranges to form an N-azo derivative. A reducing agent such as hydroxylamine produces a colorless hydrazone which, in the presence of an oxidizing agent, forms a relatively stable red N-azo derivative.

In a mechanism suggested by the present investigations thymine in carbonate solution forms a 2-keto-6-enol tautomer in which the 3-hydrogen is acidic by virtue of its position with respect to the keto group. Thymine then couples with *p*-sulfobenzenediazotate at the 3-position by a condensation process to form a yellow sulfobenzeneazo-3-thymine derivative. This N-azo compound in the presence of sodium hydroxide changes into the red quinoid form in the manner of a pH-sensitive indicator dye, and the negative valence of the sulfonate group increases from one to three. The yellow and red forms are in equilibrium with the former predominant in carbonate and the latter in sodium hydroxide. The newly proposed mechanism is in harmony with observations made on the diazo-coupling and the color-development reactions for thymine.

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BOOK REVIEWS

MESONS—A SUMMARY OF EXPERIMENTAL FACTS, by Alan M. Thorndike. 242 pages, diagrams, illustrations, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $5.50.

Mesons supplies a much needed compilation of available knowledge concerning mesons and phenomena associated with them. It is written with great clarity and the author has followed a practice which might well be more extensively used, the practice of devoting a couple of pages to definitions of symbols used throughout the text. Another very valuable feature is a glossary of terms which, in itself, forms a very useful summarization of many of the characteristics and phenomena dealt with at greater length in the text itself.

The book is so rich in the number of topics treated, that in a review one can do little more than refer to the chapters and their main contents.

Chapter 1. "Evidence of the Existence of Mesons" contains a discussion of nuclear particles in general, and embodies the essentials of the basic phenomena governing the behavior of such particles. Thus there is an excellent digest of the Bethe-Heitler theory of energy losses of charged particles. There is also one of the clearest expositions which this reviewer has seen of the meson theory of nuclear forces in elementary form. The chapter also contains experimental results.

Chapter 2. "Properties of Cosmic Ray Mesons" is a further exposition of the properties of elementary particles with a discussion of such things as mass, charge and spin. In the discussions of the evidence, the author occasionally gives very clear condensations of the analytical processes involved in the transition between experimental data and the inferences drawn from such data. Thus, for example, in speaking of the analysis of bursts, he gives a description which we quote here, not for the purposes of the phenomena themselves involved, but as an illustration of the type of condensation cited. The author writes as follows:

> One first calculates theoretically the probability that a meson of a given energy produces a high-energy electron or photon in passing through matter. One then calculates the probability that the electron or photon produces a shower of a certain size in the material surrounding the sensitive volume of the ionization chamber. One then estimates the expected ionization (and hence size of burst) to be expected by such a shower. One then takes information on the frequency of incident high-energy mesons, sums over this spectrum, and obtains a theoretically calculated distribution for the frequencies of bursts of various sizes, which can be compared with the observed frequency distribution. Many steps in the theoretical calculation are somewhat uncertain, especially the first two, so that it is by no means sure that the resulting conclusions would be reliable even if the experimental data were perfectly clear-cut and highly accurate.

Chapter 3. "New Types of Mesons." The chapter refers to the theory of the Yukawa particle, the λ-meson hypothesis and introduces the experimental facts and phenomena leading to an understanding of the heavy mesons.

Chapter 4. "Artificially Produced Mesons" deals with the various matters pertaining to the artificial production of mesons, the dependence of such production upon bombarding particle energy and other relevant parameters, a discussion of the masses of artificially produced mesons, etc.

Chapter 5. "Decay of Mesons" deals with the discussion of such decay and with direct measurements of life-times of the various particles.

Chapter 6. "Interaction of Mesons with Matter" discusses electromagnetic interactions and the scattering of mesons by matter. There is also a discussion of nuclear interactions of μ-mesons, π-mesons and so-called heavy mesons.

Chapter 7. "Production of Mesons in Cosmic Radiation" deals with the production of penetrating particles by non-ionizing radiation, penetrating showers, penetrating shower particles from air, showers and mesons from stars.

Chapter 8. "Occurrence of Mesons in Cosmic Radiation." This chapter is written, to some extent, independently of considerations discussed in earlier chapters, and welds the properties of mesons into the general scheme of the cosmic radiation as a whole. There is a discussion of such matters as meson intensity at sea level, the positive excess, the energy spectrum of mesons, meteorological effects, zenith angle dependence, geomagnetic effects, the occurrence of mesons below ground. There is a further discussion of many of these matters for high altitude observations.

The book is rich in experimental data, and this feature alone will render it of considerable value to one working in the field. Many of the chapters conclude with a summary of contents, and such summaries are very helpful. Each chapter, moreover, contains a compilation of references pertaining to the subject matter dealt with. Naturally, there are occasions where such references are incomplete, particularly where a discovery made in earlier years has received a new impetus of life at some later period. However, the literature of cosmic rays and allied phenomena is extensive, and it is not easy to attain 100 per cent perfection in the citations.

In conclusion, the author is to be congratulated upon the production of a book much needed at the present time, and one which will be of inestimable value to those working in the fields with which it is concerned. W. F. G. SWANN

FOUNDATION ENGINEERING, by Ralph B. Peck, Walter E. Hanson and Thomas H. Thornburn. 410 pages, diagrams, illustrations, 16×24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $6.75.

The design of foundations for buildings, bridges, dams and various other super-surface structures has until recent years been largely an art rather than an engineering science. True, design procedure had been developed through the years for various types of foundations, depending upon subsurface formations and conditions. But with the rapid development of the science of soil mechanics in the past twenty years, foundation design is quickly becoming a science. Of course, experience and shrewd judgment still play an important part in foundation engineering, but it is now possible to compute many factors that a few years ago were largely obtained by guessing.

Foundation Engineering, by R. B. Peck, Professor of Soil Mechanics, T. H. Thornburn, Professor of Civil Engineering, both of the University of Illinois, and W. E. Hanson of the Illinois Division of Highways, reflects the recent advances in the scientific design of foundations, and is another significant book in the broad field of civil engineering. The book has been conveniently and logically divided into four parts: Properties of subsurface materials, types of foundations and methods of construction, selection of foundation type and basis for design, and structural design of foundation elements. In the first part are identification and physical properties of rocks and soils, techniques of subsurface investigation, character of natural deposits, and a program subsurface exploration. In the second part are general discussions (no design data) of excavating, bracing, drainage and stabilization, shoring and underpinning, and of such types of foundations as footings, rafts, piles, piers, retaining walls and abutments. In the third part use is made of the science of soil mechanics to show specifically how various kinds of foundations are designed for various kinds of subsurface soil conditions. Typical data sheets are shown to illustrate standard design procedures. Foundations on sand, clay, silt, loess and on nonuniform soils are discussed and methods of design illustrated for each type. In the fourth and final part, the detail design of such foundation elements as columns, wall footings, rafts and retaining walls and abutments is illustrated.

Although elementary—and at times some of the statements are overwhelmingly obvious—the book is nevertheless comprehensive; all the fundamentals for an understanding of foundation are clearly presented. Unusual foundation problems are not treated, but such problems never are in an elementary text. To one who has only read of important men of science but never seen them this book should be of interest for it contains pictures of the significant men in the history of foundation engineering. E. W. HAMMER

METHODS OF APPLIED MATHEMATICS, by F. B. Hildebrand. 523 pages, diagrams, 14×22 cm. New York, Prentice-Hall, Inc., 1952. Price, $7.75.

To broaden the mathematical skill of the engineer and physicist, Professor Hildebrand of the Massachusetts Institute of Technology has prepared a book dealing with several fields of mathematics. The fields covered are treated at a level which requires previous experience with advanced calculus. The four fields selected embody: first, matrices, determinants and linear equations; second, the calculus of variations; third, difference equations; fourth, integral equations. These major divisions are covered by four long chapters which employ a large number of illustrative examples.

That this volume is intended for practical application is stated in the Preface and implied by the title which uses the words "applied" mathematics. At least two things are necessary to achieve the aims set forth. These are directness and appropriateness. To obtain directness the presentation has been made in an efficient manner through a strong disclosure of basic principles and the elimination of topics concerned with the classical point of view. The work has been perhaps less successful in realizing the second objective, appropriateness. A method of presentation which would appeal more to engineering professionals could have been better realized had the author considered more fully the implication of "applied" mathematics.

It has been the reviewer's observation that interesting and satisfactory problems for demonstrating mathematical principles and techniques can usually be found in electrical or mechanical engineering practice. Very frequently this matter of selecting meaningful examples is the stumbling block for the mathematical text writer. His preoccupation with classical mathematics and the vast literature on mathematical techniques has usually prevented his acquiring and maintaining familiarity with the current problems of other sciences. For illustrative examples he therefore resorts to timeworn calculations not identified with any physical problem. A real effort to overcome this failure to employ appropriate and meaningful examples would result in wider acceptance of mathematical texts among other professions.

In writing his book Professor Hildebrand has faced this situation perhaps unconsciously, for certain sections of his book contain excellent illustrative material while others have no clear application. The chapter dealing with difference equations and to a lesser extent the sections on integral equations and the calculus of variations carry illustrations with physical significance.

The reviewer is confident that the material selected for this volume is pertinent to the needs of modern engineering and that numerous interesting illustrative problems can be found which have a bearing on the subjects presented. The critical evaluation would simply indicate that since the emphasis is on "applied" mathematics, more physical significance should be worked into the problems of which there is such a large and comprehensive selection.

C. W. HARGENS

FERROELECTRICITY, by E. T. Jaynes. 137 pages, diagrams, 15×23 cm. Princeton, Princeton University Press, 1953. Price, $2.00 (paper).

Ferroelectricity is the first book, apparently, in a new series called "Investigations in Physics," edited by Eugene Wigner and Robert Hofstadter, and published by the Princeton University Press. This first volume is a revision of Dr. Jaynes' doctoral thesis, based on a theoretical study of ferroelectricity done at Princeton under the guidance of Professor Wigner.

During this study, Dr. Jaynes developed an electronic theory, as opposed to the more usual ion displacement theories, to account for the ferroelectric behavior of barium titanate and related compounds. The high polarizability necessary for ferroelectricity was attributed to low-lying electronic levels associated with the oxygen octahedra surrounding the titanium atoms. The relationship between the polarization of these octahedra and the internal fields acting upon them was developed quantum mechanically. The polarizabilities of the other ions was considered in the usual fashion. The relation between the external fields and internal fields was determined by a detailed consideration of the Lorentz factors applying to each lattice

site. The whole was then combined to give the desired relation between the applied field and the observed polarization.

The exposition of this theory forms the central core of the book. It is preceded by a review of the experimental behavior of various types of ferroelectric materials, and of the theories already put forward to account for their properties. The last three chapters contain rather detailed consideration of the mathematical techniques used in developing the electronic theory, or any theory of ferroelectricity, for that matter. A short chapter describes the purely phenomenological approach as used by Mueller and by Devonshire. The calculation of the internal fields at any given lattice site is treated thoroughly and rigorously. The general features of the relation between the internal field and the polarization are discussed from a quantum mechanical viewpoint.

From an over-all standpoint, the book contains what one must expect from a good theoretical Ph.D. thesis. It will be of great interest to other theoreticians in this and related fields, both for its exposition of the electronic theory and for its detailed treatment of some of the mathematical apparatus common to any theory. It is too specialized to be of much interest to the general worker in or out of ferroelectricity. ALAN D. FRANKLIN

HIGH SPEED PHOTOGRAPHY, by George A. Jones. 311 pages, illustrations, 15×22 cm. New York, John Wiley & Sons, Inc., 1953. Price, $6.50.

In his introduction the author writes, "The main problem in recording all fast movement is the achievement of a sufficiently short exposure time. It is generally understood that the only way of avoiding movement in a photograph is to reduce the exposure so that the amount of movement of the subject during the short time involved is insufficient to produce a noticeable degree of blurring in the record." For the next 300 pages the author explains in detail the mechanics of just how such records are made.

It is most unfortunate, but, nevertheless true, that wars spark development and research. Such is the case in the tremendous strides made in high speed photography. World War II gave extraordinary impetus to the development of this branch of photography for both in the United States and abroad—principally in Germany and Great Britain—high speed equipment for flash and movie were developed to analyze the hitherto unknown problems which arose in the laboratories.

This type of photography was highly important in the study of explosions, flame propagation, fluid flow and even air flow in wind tunnels. The application of this new device to ballistic analysis and even rocket motor action permitted notable achievements in these fields.

The author, using a remarkably lucid style in eleven well integrated chapters, describes just how this field developed and carries it through to techniques and applications in industry. The production of short flashes, the mechanism for recording during these flashes, the physical development of the picture, still photography, motion picture photography and the scientific applications are all treated fully and with a thoroughness indigenous to English authors.

High Speed Photography contains a wealth of pictures and diagrams which supplement the text. Even wiring diagrams and circuits are included to permit a clearer understanding of the problems under discussion.

At the end of each chapter is found a bibliography whose value could have been enhanced if the references were correlated with numbers in the text. An invaluable appendix lists the high speed cameras on the market and their rates, flash tubes and formulae recommended for this branch of photography.

High Speed Photography fills a wide gap in the library of the researcher, laboratory technician and scientific photographer whose field demands the application of this new development.
I. M. LEVITT

FROM LODESTONE TO GYROCOMPASS, by H. L. Hitchins and W. E. May. 219 pages. illustrations, plates, 14×19 cm. New York, The Philosophical Library, 1953. Price, $4.75.

Few discoveries have played a more influential part in the life of man than that of the compass, which is probably the oldest scientific instrument in common use today. People are

apt to think of the compass as consisting of a magnet on a pivot, and that any variation in type can only consist of a difference in size, from the pocket compass to the ship's compass. It is another common error to imagine that any compass will suit any service, whereas almost every type of ship, aircraft, or fighting vehicle calls for a compass of special type.

Several countries from China to the Baltic States claim the honor of having invented this immensely useful instrument, but the evidence of most is based upon assumption rather than on fact. The claim that the compass was used in China two thousand years before the Christian era rests upon the belief that everything old originated in that country until the Russians took up the challenge. However, there is no evidence that any magnetic principle was involved in the early Chinese direction indicators. What seems much more probable is that the magnetic compass was invented independently in both the East and the West. From the time of its discovery the compass underwent a series of slight modifications as knowledge of terrestrial magnetism improved until, in 1870, Lord Kelvin produced a compass and binnacle which was superior to all others and won universal acceptance.

The steel construction and extensive fitting of electrical instruments on the ship's bridge greatly increased the difficulty of using the magnetic compass, and this was supplemented by the need for an instrument that would furnish a quick reading of his course for the airplane pilot. The requirements of a new age promoted the development of the gyrocompass, the inductor compass, the sun compass, the compass for grid-steering, and others. Hitherto there has been no comprehensive book on the subject but the book under review, which is neither technical handbook nor complete treatise, fills a conspicuous void in scientific literature. The authors are two British naval officers who have each had about a quarter of a century's experience in the Compass Department of the Admiralty.

The first half of the book, presumably Commander May's contribution, is an excellent account of the magnetic compass through the ages, in which oft-repeated tales are subjected to the cold, calculating scrutiny of a practical navigator with a thorough knowledge of terrestrial magnetism. The second half of the book is devoted to an examination of twentieth century developments, bringing the story down to the cathode ray compass, which is still in the experimental stage. No technical knowledge is required to master the accounts given of the various later compasses, which are viewed with impartiality so that shortcomings are bared as well as advantages noted. The book should be in the library of everyone remotely interested in the compass, and it is capable of inspiring an interest where none now exists. It is an admirable introduction to the subject. T. C.

SERVOMECHANISM ANALYSIS, by George J. Thaler and Robert G. Brown. 414 pages, diagrams, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $7.50.

Essentially a general text, *Servomechanism Analysis* is intended for use in a first course on feedback control systems at the senior or graduate level. It is admirably suited for this purpose, since the authors have used a style of writing uniformly readable and understandable. In addition, involved mathematical derivations, detailed solutions of specific examples, and lengthy presentations of empirical methods and data have been reduced to a minimum. The student is left to investigate subtle details for himself by solution of the problems given at the end of each chapter and by referring to the many listings in the bibliography.

There is nothing in this book which represents an original approach to either system analysis or design. The development of such subjects as the use of the Laplace transformation and transfer functions, stability criteria, compensation techniques, and the logical order in which these subjects are taken up must certainly be familiar to those who have read the various other texts in this same field. However, some features of the book are new in that certain material is presented which has not previously been included in a first course text. The authors have acknowledged the increasing importance of Relay Servomechanisms and The Root-Locus Method of system analysis by including an introductory chapter on each of these subjects. Discussions of the nature of the physical components which the engineer must use to build his control systems are appropriately included throughout the text. However, these discussions are necessarily brief, serving merely to justify the mathematical expressions used

to represent the components in sample problems. The actual construction details and principles of operation of widely used system components, as well as some of the difficulties encountered in their use, are discussed in four appendices.

Since the book has been written by, and is obviously primarily intended for, electrical engineers, the majority of the discussions, examples, and problems are given in terms of electrical circuits and electrical machinery. The symbols and notation used are those which are most familiar to electrical engineers. The appropriateness of this should not, perhaps, be discussed here; however, there are those who will undoubtedly criticize the book on these grounds in view of the efforts of many, including the AIEE Subcommittee on Symbols for Feedback Control Systems, to encourage the adoption of a standard notation which will be acceptable to all engineers who are engaged in the many different branches of automatic control work.

G. H. FRIEDMAN

ENCYCLOPEDIA OF CHEMICAL REACTIONS, Vol. V, edited by C. A. Jacobson. 787 pages, 16×24 cm. New York, Reinhold Publishing Corp., 1953. Price, $15.00.

The present work—Vol. V in the series—was completed after the death of Dr. Jacobson who spent so much of so many years compiling this *Encyclopedia*. It is certainly to be regretted that he could not have lived to see such a monumental and useful work as an entity. The work is to be carried on in succeeding volumes by an associate editor, Clifford A. Hempel.

Volume V continues the cataloguing of inorganic reactions by elements, from Nickel to Ruthenium. The advantages of this form of recording of inorganic chemistry for reference usage become obvious on perusal. Under a given element, a separate page wide section is given to each reaction in which the element is known to be involved. The reactions are alphabetized primarily by the compound in which the element occurs, such as NH_2Cl, NH_2OH, etc., for nitrogen. The secondary alphabetization is by the compound with which the primary reference compound reacts. Thus, under NH_2OH, we have I_2 NH_2OH, O_2 NH_2OH, etc. It is thus relatively easy to locate any reaction listed without recourse to the voluminous and complete index in the back. These indices consist of "Index to Reagents" and "Index to Substances Obtained."

One wonders a little whether the task of indexing, in the latter case, has not been taken too literally. It is perhaps somewhat difficult to see the usefulness of the listing in which references are given to almost 500 reactions in water, for example.

This volume is clear, concise, readily used and undoubtedly reasonably complete. There is no doubt that the set will be a classic in the realm of chemical reference books.

ALBERT L. MYERSON

ANNUAL REVIEW OF NUCLEAR SCIENCE, Vol. 2. 429 pages, diagrams, 16×24 cm. Stanford, Annual Reviews, Inc., 1953. Price, $6.00.

Presenting 16 lengthy papers on subjects in which knowledge has been significantly advanced in recent years. this volume includes such diverse topics as cosmic rays, β-decay, fission, scattering, reactions, moments, radiocarbon, mesons, isotopes, accelerators, radiation effects in solids, distribution of elements, and stellar energy production.

These review articles generally give a critical discussion of recent experimental and theoretical papers and attempt to integrate their results in a coherent picture. Various theories are presented and analyzed. Tentative conclusions as well as the unsolved problems are stated. These articles reveal that nuclear science has made great progress in understanding the basic facts, but, at the same time, the reader is strongly impressed with the fact that nuclear physics is on the boundary of our domain of knowledge and that a great deal is not known. These annual reviews serve a valuable purpose in collating the great volume of research that is currently pouring forth in our journals and in giving the non-specialist a good view of the achievements in nuclear science.

Most of the articles are excellently written with clear statements of the topic being surveyed and concise summaries of what has been shown in the body of the article. Some of the papers are actually short monographs and are the first reviews in these fields. The literature

surveys ended in early 1952 and usually concentrated on recent years; however, some (for example, the chapter on accelerators) restricted themselves to the year 1951, because of other fairly recent reviews. The papers are well documented with the number of references per paper being between 50 and 200. The chapter on cosmic rays comes to the conclusion that cosmic rays are mostly of stellar origin. No mention is made of the recent work of Pomerantz and McClure who indicated that the sun is chiefly responsible for our low energy cosmic rays, presumably because this paper appeared after the conclusion of the survey.

A slight confusion may arise from the inclusion of the 1954 table of contents just ahead of this year's. The volume is attractively bound but the paper could be of higher quality.

LEONARD MULDAWER

ADVANCED MATHEMATICS IN PHYSICS AND ENGINEERING, by Arthur Bronwell. 475 pages, diagrams, 16×24 cm. New York, McGraw-Hill Book Co., 1953. Price, $6.00.

This is a text in mathematical physics written for the advanced undergraduate or beginning graduate student. The mathematics covered consists essentially of the following: Fourier series and Fourier integrals, ordinary and partial differential equations, vector analysis, functions of a complex variable including conformal mapping, and Laplace transformations.

The chapters on ordinary differential equations consider mainly linear equations. However, there is considerable discussion of the solution of differential equations in series, leading to the theory of Bessel and Legendre functions.

The main feature of the treatment of partial differential equations is a unified discussion of the most important partial differential equations of mathematical physics. These are equations of second order, including Laplace's equation, the diffusion equation, and the wave equation. The approach used depends on the method of separation of variables, which reduces the partial differential equation to ordinary differential equations. The theory of partial differential equations is then applied to problems in heat flow, fluid dynamics and electromagnetic theory.

The development of the theory of functions of a complex variable leads to the consideration of two main topics, that is, conformal mapping and dynamic stability. Conformal mapping is used to obtain the solutions of Laplace's equation in two dimensions. The Routh-Hurwitz and Nyquist stability criteria are developed. These have wide applications in electric circuit problems and in aerodynamic problems.

The text includes many completely worked out illustrative problems. There is a bibliography and a set of problems (with answers) at the end of each chapter.

A fairly complex text on the subject at this level, this would also make a very satisfactory reference book.

H. L. PLATZER

MECHANICS OF MATERIALS, by Seibert Fairman and Chester S. Cutshall. 420 pages, diagrams, 16×24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $5.75.

Normally, another book on mechanics of materials or strength of materials is hardly cause for excitement; it seems as though a new one comes out every month. However, this book on the subject is different from the average, not so much in the inclusiveness of the material but rather in the comprehensiveness with which the included material is treated. Deliberately written for the average rather than the superior student, and reflecting the many years of the authors' teaching experience at Purdue University, the subject matter is presented in such a fashion that it seems to be quite easily understood. Well illustrated and containing a number of worked out representative examples as well as any number of problems (with answers), the book should be a very useful text for the beginning engineering student.

Introducing the subject with a discussion of the fundamental concepts of stress and strain and elasticity, as is customary, the authors proceed to the usual subjects contained in such books. The mechanical properties of materials, riveted and welded joints, thin cylinders and spheres, torsion and shear, bending and stresses in beams are next described. Considerable space is then devoted to the deflection of beams by the double integration and area-moment methods, particularly the latter. Following a discussion of statically indeterminate beams,

the authors have an interesting chapter on such special beams as those of two materials, of reinforced concrete, of variable cross section, and of materials not conforming to Hooke's Law. Columns and combined stresses are next treated. The concluding chapter on strain energy and impact is very well presented and very significant. The method of determining the magnitude of the impact load in the case of axial load, shear, bending and torsion is very clearly presented and explained by illustrative examples. For anyone who has wondered what load is developed in a beam or torsional shaft under a falling weight, he need only study this section. E. W. HAMMER

SELECTED TOPICS IN MODERN INSTRUMENTAL ANALYSIS, edited by David F. Boltz. 477 pages, illustrations, diagrams, 16×23 cm. New York, Prentice-Hall, Inc., 1952. Price, $6.00.

Designed for an introductory course in chemical analysis by instrumental techniques, this text makes no claim to completeness but rather aims briefly to outline the more common instrumental methods in use in modern analytical laboratories. Each of its ten chapters is devoted to one method. Six of the chapters were contributed by industrial analysts while the editor prepared two of the four university contributions.

The methods dealt with are: Electrometric pH Measurement, Polarography, Fluorometric Analysis, Spectrophotometric Analysis, Spectrochemical Analysis, Electrometric and Opticometric Methods (a miscellaneous chapter prepared by the editor), X-ray Diffraction, Mass Spectrometry, Raman Spectroscopy, and Radiochemical Analysis.

The emphasis is definitely on technique as used in industrial and control laboratories rather than on the physical principles underlying the various methods. As a matter of fact, discussions of theory are limited to the barest minimum necessary for an understanding of the method. Each chapter describes in some detail the construction and operation of the necessary equipment. Some chapters in addition describe rather fully the various commercially available instruments. Errors to which each method is subject are described along with principal fields of application. Methods for interpreting data are outlined.

A few problems, questions and exercises are scattered throughout the text with more given in the appendices. Tables of Redox potentials, solubility products, etc. are also found in the appendices. Each chapter contains its own bibliography. E. R. STEPHENS

THE PRINCIPLES OF LINE ILLUSTRATION, by L. N. Staniland. 212 pages, illustrations, 15×22 cm. Cambridge, Harvard University Press, 1953. Price, $5.00.

Illustrations and diagrams are often extremely valuable in clarifying reports on research work, with a consequent lessening of the amount of written description. The man who has to write the report often fails to give serious consideration to the suggestion that he illustrate his work with drawings on the plea that he is no artist. This book has been written for just such a man. It makes no pretense to make an artist out of him, but it does provide instruction on the choice of materials and the methods by which anyone lacking a sense of proportion and shape can produce line drawings suitable for reproduction. The emphasis is entirely upon practical aspects, and the reader is not offered any unnecessary art theories.

The author is a biologist and the drawings chosen to illustrate his methods are of biological specimens, but this in no way detracts from the description of the ways by which the effects are attained. These effects can be applied to drawings on any subject. The chapter on the techniques of line drawing should be of interest to everyone who desires fundamental instruction, and the final chapter contains some hints on such things as making improvised lantern slides of diagrams, the making of negative drawings, the bleaching of photographs, etc.

Within the limits he has set himself the author has prepared a very useful aid to the would-be draftsman. T. C.

HEAT TRANSFER PHENOMENA, by R. C. L. Bosworth. 211 pages, diagrams, 15×22 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.00.

The limited number of modern works on the subject of heat transfer phenomena makes any new addition to the list welcome. This is especially true when its author is someone as

well qualified by training and experience as R. C. L. Bosworth. His ability to clarify this neglected and difficult subject greatly increases the value of his book.

In a field of this type where the amount of research is small and the experimental difficulties are great, the amount of data available is of necessity limited and the correlations of these data largely empirical. Thus the author's approach to the problems of heat transfer through the other related transport processes is very helpful to a better understainding of the mechanisms involved.

Dr. Bosworth's book is well organized and can serve as a fine introductory work as well as a reference handbook. It contains a full discussion of all the various heat transfer processes, conduction, radiation, and forced and natural convection. There is also an excellent theoretical section on the transport processes as a whole.

As an experimental tool, the concept of the equivalent electrical circuit is introduced and fully discussed. By the use of this approach, many long duration heat processes can be studied in a reasonable time under laboratory conditions. The same method may be modified to apply to rapid heat transfer problems in order to increase the time to a more convenient scale.

The theory of thermodynamical similarity is discussed with regard to the use of models for the understanding of full scale processes.

In conclusion, one can state that this book will be of great value to anyone working or interested in the field of heat transfer phenomena. It serves to collect and coordinate the work which has been done on this subject during the period 1940 to 1951. In this respect, it stands alone, at least at present, in the literature of modern heat transfer theory.

H. BICKFORD

THE CHEMISTRY OF SYNTHETIC DYES, Vol. II, by K. Venkataraman. 1442 pages, diagrams, 16×24 cm. New York, Academic Press, Inc., 1952. Price, $10.00.

The second and final volume of the monograph on the chemistry of synthetic dyes continues and completes the detailed survey of the dyestuffs used in the industry, providing a wealth of detail for the dye chemist. In it, methods of preparation of each of the commercially available dyes are described, and comparisons made between dyestuffs of similar composition but differing by some slight modification for specific uses. More than 1200 of the 1400 pages of this volume are devoted to the chemistry and uses of the synthetic dyes, and the description of the chemical processes are amply substantiated with references.

Following these are chapters discussing the action of light on dyes and dyed fabrics, the nature of dye absorption, identification and evaluation of dyestuffs. There is an interesting discussion of the complications involved in the quantitative assessment of fading. The somewhat empirical tests of the industry are given in detail, and many variables that enter into such an assessment are reviewed. The theories concerning the mechanism of fading are given in broad outline. Theories attempting to explain why certain dyes are more readily absorbed on specific fibres are discussed, and generally a correlation is sought between the structural features of the dye molecule and its affinity for the specific fibre.

The problems of identification are reviewed, and the commerically available methods described. Considerable emphasis is given to the chromatographic methods. The thoroughness and completeness of this monograph recommend its use by chemists interested in the chemistry of synthetic dyes.

S. MUCHNICK

RECURRENT ELECTRICAL TRANSIENTS, by L. W. Von Tersch and A. W. Swago. 399 pages, diagrams, 15×22 cm. New York, Prentice-Hall, Inc., 1953. Price, $7.75.

Recurrent Electrical Transients is another fine book in the Prentice-Hall Electrical Engineering Series. It is a credit, therefore, to the authors and to Dr. W. L. Everitt, editor-in-chief of the Series.

This new text book is concerned with the analysis and synthesis of circuits in which the steady state wave forms are considered as series of recurring transients. Such wave shaping circuits are the building blocks or basic component circuits of many electronic devices. Analog

and digital computers, radar and television systems are but several examples where the techniques described in this book are useful.

The material in the nine chapters is well organized and logically presented. Time analysis and time constant concepts are used instead of frequency response concepts, an approach which has been found more useful to electronic circuit designers, yet few texts use it.

The lead chapter introduces fundamental concepts of resistance, inductance, and capacitance as circuit components. The nonlinear aspects of resistance, as exemplified by vacuum and gas tube characteristics and those of germanium diodes, are brought in here, and analyzed in the second chapter, which deals also with the response of RLC circuits to a unit step with various initial conditions.

The next three chapters consider: the response, in terms of the time constant concept, of networks to simple wave forms; clamping; and clipping. Diode and triode circuits are featured in these analyses with reference to their use in television systems. Sweep circuits, synchronization and linearity problems associated with electrostatic and magnetic deflection cathode ray tubes are treated next. Nonlinear characteristics and requirements are analyzed. The last two chapters are devoted to analysis and design of trigger circuits—for example, Eccles-Jordan and scalers—and of multivibrators. The latter includes plate and cathode coupled circuits and single cycle and free running systems.

The many illustrative problems used throughout the text help the reader to understand the material. While these are supplementary, this reviewer thinks that for such a book they are an absolute necessity, as are the many analysis and design problems at the end of each chapter. The reference bibliography, part of which is included in each chapter, spans the period from at least 1917 to 1950.

If another edition of this book is to be printed, more careful proofreading would be desirable, to eliminate some glaring errors which seem to occur more frequently in the early part of the volume.

Practicing engineers who have not had formal training in pulse circuits would do well to read this book for its pedagogical value. S. CHARP

BOOK NOTES

BASIC WELDING PRINCIPLES, by Emanuele Stieri. 220 pages, illustrations, diagrams, 22×28 cm. New York, Prentice-Hall, Inc., 1953. Price, $4.30.

Following the recommendations of the American Welding Society, this well printed and profusely illustrated "handbook" of welding techniques can be used either by the beginning welder or the expert. It provides detailed descriptions of tools and methods, complete with procedures for almost every conceivable metal which can be welded. The artist—John G. Marinac—is to be congratulated on his exceptionally clear drawings. The author's style is simple and lucid and his final product is a definite contribution to welding literature. Experienced welders will find much that is new to them, and the beginner may teach himself by a careful study of the text and by performing the progressive practice exercises provided.

AN INTRODUCTION TO LINEAR PROGRAMMING, by A. Charnes, W. W. Cooper and A. Henderson. 74 pages, 21×28 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1953. Price, $2.50 (paper).

The two major parts of this work are: Part I, An Economic Introduction to Linear Programming (by Cooper and Henderson); and Part II, Lectures on the Mathematical Theory of Linear Programming (by Charnes), originally developed as part of a research seminar at Carnegie Institute of Technology. Those interested in applications will be helped by Part I; those who wish to learn the theory and special aspects of the subject will find these in Part II. Part I has been cross-referenced to Part II, to provide a convenient method of linking theory and applications. The relatively inexpensive volume should prove of value to economists, engineers, statisticians and mathematicians as well as to those in business and industry.

GENERAL COLLEGE CHEMISTRY, by Frank Brescia. 581 pages, diagrams, 16×24 cm. New York, The Blakiston Co., Inc., 1953. Price, $6.00.

Intended as a first year college course for students in science and engineering, this new text includes much descriptive material, integrated with principles, general concepts, theories and applications. Many of the 30 chapters have appendices containing additional material for use as the teacher sees fit; all chapters are followed by several problems, most of which have answers given, and most of which are of a quantitative nature, depending upon an understanding of principles for their solution.

PHOTOELECTRIC TUBES, by A. Sommer. Second edition, Methuen's Monographs on Physical Subjects. 118 pages, diagrams, 11×17 cm. New York, John Wiley & Sons, Inc.; London, Methuen & Co., Ltd.; 1952. Price, $1.90.

The second edition of this useful monograph has changed its title to *Photoelectric Tubes* instead of the original *Photoelectric Cells*, to emphasize the fact that the book is concerned primarily with electronic tubes based on photoelectric emission, but does not treat photovoltaic and photoconductive cells. A new chapter on the theories of photoelectric emission, by S. Rodda, replaces the elementary treatment of this subject in the first edition, and many other discussions have been brought more nearly up-to-date. The bibliography has been expanded, with 21 out of a total of 69 references dated between 1946 (the date of the first edition) and 1951.

THERMIONIC VACUUM TUBES AND THEIR APPLICATIONS, by W. H. Aldous and Sir Edward Appleton. Methuen's Monographs on Physical Subjects. Sixth edition, 160 pages, diagrams, 11×17 cm. New York, John Wiley & Sons, Inc.; London, Methuen & Co., Ltd.; 1952. Price, $2.00.

Material progress in the development of thermionic vacuum tubes and their use as circuit elements made it necessary to revise completely the 1931 edition of this monograph. The radical revision of Appleton's original work was made by W. H. Aldous, who has kept in mind the audience for which the first edition was intended—"the student of general physics who has not made a special study of radio-frequency phenomena." By bringing the material up-to-date, the monograph will serve its readers more effectively. Sixty-five of the 119 references are dated 1936 or later, with 34 of these dated between 1945 and 1950.

LABORATORY MANUAL OF MATERIALS TESTING, by R. T. Liddicoat and Philip O. Potts. 239 pages, diagrams, illustrations, 14 × 21 cm. New York, The Macmillan Co., 1952. Price, $4.00.

Prepared from lecture notes and laboratory experience, this useful manual is the result of some ten years' experience in Materials Testing courses at the University of Michigan. The manual is intended to supplement textbook study of the strength of materials and to emphasize the properties of engineering materials. In general, tests are set up under the following headings: Purpose, Equipment, Procedure, and Requirements. A wide range of tests is covered, including hardness testing, strain gages, metal grain structure, tension testing, shear and torsion testing, photoelastic testing, fatigue and impact testing, etc. This manual should prove to be a practical help in one-semester engineering courses.

CURRENT TOPICS

Robot Psychologist.—A "robot psychologist" with an electronic brain, memory, and "show window" viewing screen is helping human psychologists "see" that the right man gets the right Army job.

The huge two-and-one-half ton, double-section electronic computer is known as a psychological matrix rotator. It was developed by engineers at the General Electric Company's General Engineering Laboratory, Schenectady, N. Y., from a basic design prepared by Dr. Richard H. Gaylord, prominent Department of Defense psychologist. It is being used by the Personnel Research and Procedures Division of the Adjutant General's office.

Described as so complex that only skilled psychologists can operate it and understand results, the machine was in planning and consideration for three years at the Personnel Research Branch and an additional two years in research and development at the G-E laboratory. George T. Jacobi, G-E analog computer engineer, said continuous collaboration between engineers and psychologists resulted in a computer "optimally suited to the psychologists' needs."

Psychological problems are first translated into statistical form and then fed into the giant computer. A maze of vacuum tubes, potentiometers, wires and other electrical apparatus literally hums through the data and flashes results on a cathode ray tube, similar in appearance to a 16-in. television viewing screen.

The operating psychologist sees the test results in the form of "scatter diagrams" or dot patterns. Data not required during a particular problem are stored in the machine's electronic "memory" for future use.

G-E engineers say time required for solving complex problems can be reduced from months or even years to a matter of days through use of the new computer.

According to G-E engineers, the psychological matrix rotator was developed to handle statistical data representing 12 different abilities underlying up to 50 tests. However, Department of Defense psychologists have already learned to adapt it to much larger problems.

The giant computer comprises two sections. The five-by-five-by-six-foot input cubicle contains electronic equipment for receiving and storing statistical data. The operator's console, approximately twice this size, but stationed six feet away, consists of controls for receiving information from the input cubicle, the computing elements, the viewing screen, and a camera for photographing "cluster" data.

At present, according to G-E engineers, the "robot psychologist" is being used to facilitate studies designed to improve current methods of mass psychological testing.

Artificial Lightning Tests.—Higher transmission voltages have created a need for experimental surge data. The incremental cost of extra-high voltage equipment calls for careful coordination of arrester characteristics and equipment insulation in order to maintain the lowest practical margin between pro-

tective levels and insulation levels. Surge characteristics at voltages at which corona is important are fundamental to this consideration.

The highly insulated experimental lines of the American Gas and Electric Company at Brilliant, Ohio,[1] are ideally suited to the gathering of such data. Not only are there transformers and lightning arresters at the site, but the transmission lines are strung with conductors of the diameter and type that will be used in extra-high voltage transmission. In addition, the high-voltage substation is typical with respect to spacing of conductors and the presence of open stub busses that could cause doubling of surge voltages.

Recognizing the excellent opportunity afforded by this experimental equipment, the American Gas and Electric Company and the Westinghouse Electric Corporation have undertaken a lightning test program. The objectives of the full-scale surge tests that are now under way in this field laboratory are:

1. to determine the attenuation of surge voltages that travel along the line
2. to determine the slope of these travelling wave fronts
3. to determine voltage coupling factors
4. to determine the influence of line conductor attenuation, in conjunction with various combinations of substation equipment, on the selection of substation insulation levels
5. to determine the effect of several lines entering the station on the lightning voltage in the station
6. to determine the most effective location of lightning arresters with respect to the remainder of the substation equipment.

Westinghouse supplied a surge generator that has been erected at the open end of the two 1.5-mile lines, farthest from the substation. It is mounted on four poles and can deliver into one conductor of a transmission line either positive or negative surges of approximately 1800 kv crest value, 0.6 micro-seconds front, and 7 micro-seconds to half magnitude. The surge can be applied to any one conductor, or any combination of conductors.

Of the two three-phase lines available, one is strung with 1.65-in. HH copper, and the other with 2-in. expanded ACSR. Both lines are insulated beyond the level required for the highest transmission voltages considered to date.

The two hot-cathode DuMont cathode ray oscillographs being used have photographic equipment capable of a writing speed of about 1000 miles per second, thus making it possible to record the fastest wave fronts. The oscillographs are coupled to the lines (or station bus) with specially-designed voltage dividers that have low burdens and small distortion.

One oscillograph located near the surge generator records the applied surges. The other is portable, and is used to measure the voltages at any point on either line, or at any point within the substation. Terminating resistors are available to approximately simulate infinitely long lines.

Many valuable data have been collected from the active testing over the past few months. It is expected that several more months of field work will be required to complete the tests.

[1] The same lines that were used for the determination of 60-cycle corona loss and radio influence characteristics in earlier tests.

Smelting Furnace Dust Recovered.—The International Nickel Company of Canada, Limited, like the ordinary housewife, has its dust problems, but with a difference. The Inco dust is a special variety containing small quantitities of nickel and copper and is produced in operations of the Company's huge smelter here. This dust is recovered at a daily rate of hundreds of tons and turned back into production instead of being allowed to escape into the air.

Recovery of the dust contributes to plant efficiency by making otherwise waste material available for production. The dust is recovered from gases from the batteries of roasters and converters in the smelter—which is the largest of its kind in the world—and is used again in the nickel reverberatories. This giant dry-cleaning operation is carried on in the Cottrell precipitation plant, which operates at such high efficiency that only a small percentage of furnace dust gets by.

On their way to the plant's 500-ft. smoke stack the gases pass into huge flues in the treater section of the Cottrell plant in which are suspended twisted-wire curtains charged with 60,000 volts of electricity. Parallel to the wire curtains are grounded 22-ft. rods which act as collecting electrodes. Receiving this big electric shock, the ionized particles of dust are precipitated on the rods and the gases pass on to the stack. Once during each shift the power is turned off in each unit of the treater section and the rods are tapped by pneumatic hammers to knock off the dust, which falls into bins. Picked up by a conveyor, the dust is transported to a shooting tank which blows it through pipes to the reverberatory furnaces where it re-enters the smelting process.

Power for the Cottrell operation is received at 550 volts a.c. and in a battery of transformers and rectifiers is stepped up to 60,000 volts d.c. On either side of each rectifier are coils which prevent interference with radio reception in the area. All openings giving access to high-tension equipment or electrodes are protected by automatic grounding switches to prevent workers from coming into accidental contact with live parts carrying high voltage.

The temperature of the gases as they pass through the treater section is watched constantly. If it falls below a certain point the result is a condensation of acid which corrodes the equipment. Heat from the reverberatory furnace boilers is used to overcome this difficulty.

Continuous Vacuum Melting Furnaces.—U. S. Patent No. 2,625,719 has recently been granted to the National Research Corporation of Cambridge, Massachusetts. The patent covers continuous vacuum melting furnaces which have been developed by the Metallurgical Department.

This new development now makes it possible to charge raw material and remove vacuum cast ingots from furnaces without the necessity of cooling the crucible or breaking the vacuum in the melting chamber. Vacuum melting furnaces are used in the production of titanium, zirconium, and molybdenum and in the production of specialty metals and alloys used where the ultimates in physical, chemical, and electrical properties are required.

Air Dampening System for Lithography.—Fifty years of the same old practices in lithographic printing may end because of a new push-button gadget, the U. S. Army's Corps of Engineers Research and Development Laboratories, Fort Belvoir, Va., announced recently.

The Air Dampening System, invented by William G. Mullen, now associated with the A. B. Dick Company, Chicago, is an air-blade that cuts water to a uniform thickness. The A. B. Dick Company is under contract with the Engineer Research and Development Laboratories to develop such an experimental air dampening system.

The Army Engineers' interest in lithography centers around a faster method of map reproduction in the field. According to Robert E. Rossell, Chief of the Map Reproduction Branch, "The Air Dampening System will simplify the process of lithography. It will eliminate delays such as those caused by the adjusting of normal dampening systems making lithography more economical." Mr. Rossell added, "the quality of the lithographic reproduction is improved."

The lithographic printing process is predicated on the premise that water and grease will not mix. The grease—ink—is applied to lithographic plates and received by those surfaces not previously dampened by water. The theory is sound and it works, but in actual practice lithographers are plagued with the necessity of keeping dampening rollers clean, adjusted, and of uniform dampness.

The conventional dampening system employs the technique of absorptive fabric (Mollerton) covered rollers having direct contact with the plate. A perfect balance between ink and water is required. During adjustments the dampening rollers pick up ink as a result of too little moisture or water gets into the ink distribution system due to excessive moisture. Once optimum dampness is achieved, continued attention is required to maintain it. This system is not economical and requires highly skilled technicians to operate lithographic presses.

With Mr. Mullen's air system, a chrome-plated steel roller is used which does not contact the lithographic plate. Set at .005 of an inch from the plate, the roller applies an excess amount of water which is cut off to a uniform thickness by the air-blade on each revolution before the plate contacts the inking system.

The new invention can be mounted on standard lithographic presses. An air stream, across the entire length of the printing cylinder set at the proper angle and pressure, removes excess water automatically when the button is pushed to engage it.

The Air Dampening System is not yet a finished product. A test model has been made, but before it goes into production, exhaustive performance tests by the Engineer Research and Development Laboratories and private industry must first be completed.

Tricolor Television Tube.—A patent covering the invention leading to the first demonstrated direct-view, tricolor kinescope (television picture tube) has been issued to Dr. Alfred N. Goldsmith, radio pioneer and New York consulting engineer, who has assigned the patent to the Radio Corporation of

America, according to the Official Gazette of the U. S. Patent Office, of March 3, 1953.

In Dr. Goldsmith's invention, which he made more than ten years ago, the viewing face of a kinescope is coated with hundreds of thousands of elements of primary color phosphors (luminous material)—red, green and blue. A full color picture is reproduced by exciting the phosphor dots with precisely controlled and rapidly scanning electron beams.

Such tubes were first shown publicly in Washington on March 29, 1950 in demonstrations of RCA's all-electronic compatible color television system. Research and engineering on tricolor kinescopes have been conducted by RCA Laboratories Division and RCA Victor Division since World War II.

Dr. Goldsmith's patent (No. 2,630,542) describes and claims the "shadow-mask" feature whereby a perforated screen is mounted just behind the picture screen. The shadow-mask has one tiny perforation for each grouping of three primary color dots. The beam for exciting the red dots, for example, is made to pass through the perforation at such an angle that a red phosphor is illuminated and the other two are shadowed and unactivated.

One form of RCA tricolor kinescope has 200,000 perforations on the shadow-mask and a total of 600,000 phosphor dots, 200,000 for each color. The individual dots are indistinguishable at normal viewing distance. Every second, the entire screen is scanned 30 times, giving a continuous smooth moving picture in color.

Photographic Typesetting Developed (*News Letter No. 32*, Armed Forces Communications Association).—Electronics has come up with a new typesetting device which sets up type on a photographic film in millionths of a second. The machine, known as "Photon," employs a standard electric typewriter mechanism, with an extra row of keys at the top, and the whole mounted in a frame something like an organ console with some button controls like the stops on an organ. "Photon" can shift from one to another type style in the middle of a line, and size can be instantly varied from 5-point to 36-point type, making the typesetting process very flexible for use in advertisements, engravings for off-set printing, etc. The first book printed on such a machine was presented to the library of MIT as the gift of the Graphic Arts Research Foundation, in February of this year.

One-Man Helicopter.—A one-man collapsible helicopter which can fly one and a half hours on gasoline, kerosine or diesel fuel oil without refueling, is powered by two pulse-jet engines mounted on the tips of two rotor blades. The tiny helicopter can be dropped from larger planes, unpacked by two men, and put in the air in about 20 minutes. It stands less than six feet high and will carry a load of more than its own 300-lb. weight at a top speed of 80 miles an hour.

Radiation Measurement Device Perfected.—The new, inexpensive, accurate "*dosimeter*," a device which measures the total amount of gamma or X-radiation to which its wearer has been exposed, has been recently perfected by the Navy. The DT-60/PD phosphor glass dosimeter weighs less than an ounce and is worn on a dogtag chain or belt clip. Its accuracy is about

20% for a wide range of radiation. Its diameter and its cost is less than a silver dollar and it will last longer than the man who wears it. The instrument will pick up and accumulate, even after many readings. Its principle: the more radiation, the more fluorescence.

400-Ampere A-C. Welding Transformer.—A new 400-ampere single-phase a-c. welding transformer with a 60 per cent duty cycle has been announced by the General Electric Company's Welding Department. Designated as G-E Type WK40K, the new welder provides a current range of 40 to 500 amperes, and can be used with a variety of electrode sizes for repair, maintenance, and construction work.

According to G-E engineers, the new transformer not only assures quick starting, but also incorporates arc-stabilizing capacitors which make it easier for operators to strike and maintain an arc without popouts. This results in faster travel speeds, fewer "patch ups," and stronger welds, they said.

The extra-wide current range allows the use of this one machine for a wide variety of applications, from light-duty, low-current sheet metal jobs to heavier-duty, high-current industrial work. A range switch enables the operator to change quickly from high to low currents or vice versa.

Other advantages of the new welder are longer coil life, because of silicone insulation, stepless current control permitting accurate adjustment, and built-in idlematic control for protection against electric shock.

The new welder is 44 in. high, 22 $\frac{5}{8}$ in. in diameter, and weighs 355 lb.

Brain Operations Aided by Transparent Instruments (*Petroleum Newsnotes*).—Transparent plastic instruments have been developed for brain surgery. Made of methyl methacrylate, which is petroleum-derived, the instruments have many advantages over their metal counterparts. They are light, inexpensive, easily made, and they have a smooth surface and rounded edges which protect the brain. Underlying cortex and white matter can be seen through the clear plastic which does not shine reflected light into the surgeon's eyes or conduct electric current. One set of instruments already has been used continuously for more than four years.

Improved Self-luminous Markers Developed.—The United States Radium Corporation has recently developed self-luminous markers using strontium-90 as the radioactive phosphor. Used for both self-luminous deck and personnel markers by the Navy, the phosphor has proved to have longer useful life, greater brightness values over radium markers and has greatly reduced health hazards. Extending the usefulness of such markers in night operations, the strontium markers can be manufactured at present in all colors except red.

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